SOLUTIONS

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

CSI 35 Exam 3
Kerry Ojakian SPRING 2026
YOUR NAME (first, then last):

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points (scaled to 100 total).

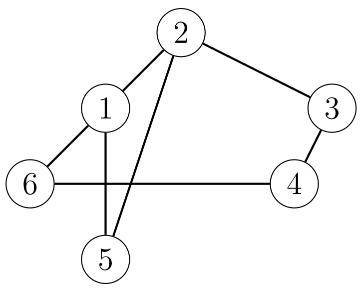
1. Consider the graph below.

   (a) How many vertices does it have? How many edges does it have?

   (b) On the graph, label the degree of each vertex.

   (c) Draw the induced subgraph that has vertices 2, 3, 5, and 6.

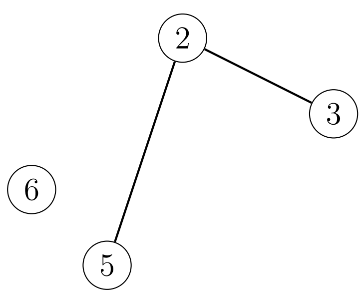
   (d) Draw any subgraph whose vertices are 2, 3, 5, and 6; but which is **not** an induced subgraph.

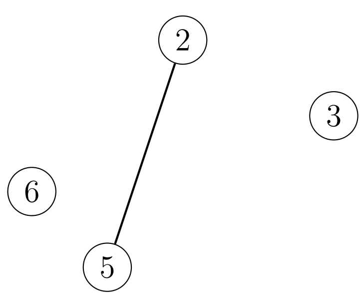


SOLUTION.

   (a) The graph has 6 vertices and 7 edges.

   (b) Vertex 1 has degree 3, Vertex 2 has degree 2, Vertex 3 has degree 2, Vertex 4 has degree 1, Vertex 5 has degree 2, Vertex 6 has degree 2

   (c) 

   (d) For Example. 

2. Draw the Hasse diagram for the divisibility poset for $\{1, 4, 8, 20, 32\}$ (i.e. (a, b) is in the poset when $a|b$). Indicate any least or greatest elements.

   SOLUTION.

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32
 |
 8   20
 |   /
 4
 |
 1
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   The least element is 1. There is no greatest element.

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3. Draw the graph $K_{4,5}$. What is its chromatic number?
   How many edges does $K_{c,d}$ have, for integers c and d?

   SOLUTION.

   $K_{4,5}$ is a complete bipartite graph with two sets of vertices; one with 4 vertices and another with 5 vertices. Each vertex in the first set is connected to every vertex in the second set. To find the chromatic number of $K_{4,5}$, observe that the chromatic number $\chi(K_{c,d})$ for any complete bipartite graph $K_{c,d}$ is 2, as it is bipartite and requires only two colors to ensure no two adjacent vertices share the same color. Thus, $K_{4,5}$ has chromatic number is 2.

   The number of edges E in $K_{c,d}$ is given by the product of the sizes of the two sets of vertices:

$$E = c \times d$$

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4. Consider the following adjacency matrix for a graph on vertex set $\{1, 2, 3, 4, 5\}$.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Draw the graph, write down its edge set, and find its adjacency list representation. What is the common name for this graph? (i.e. from among the classes P_n, C_n, K_n, $K_{a,b}$, etc)

SOLUTION.

Vertices: $\{1, 2, 3, 4, 5\}$
Edge set: $\{(1, 2), (1, 5), (2, 3), (3, 4), (4, 5)\}$
Adjacency list representation:

$$1 : \{2, 5\}$$
$$2 : \{1, 3\}$$
$$3 : \{2, 4\}$$
$$4 : \{3, 5\}$$
$$5 : \{1, 4\}$$

This graph is commonly known as C_5, a cycle graph with 5 vertices.

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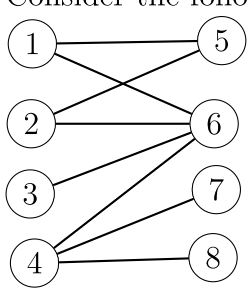
5. Consider the following graph, where $L = \{1, 2, 3, 4\}$ and $R = \{5, 6, 7, 8\}$.



Show that there is **not** a matching from L to R by finding a violation of Hall's Condition.

SOLUTION.

For example, if $X = \{1, 2, 3\}$ in L, then its neighbor set is $N(X) = \{5, 6\}$.
Since $|X| = 3 > 2 = |N(X)|$ we have a violation of Hall's condition.

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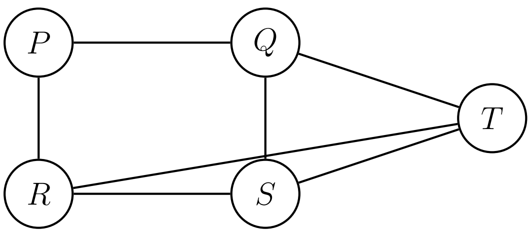
6. Consider the graph given by the following adjacency list representation. Draw the graph. What is its connectivity?

P	Q, R
Q	P, S, T
R	P, S, T
S	Q, R, T
T	Q, R, S

SOLUTION.

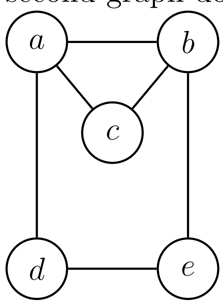


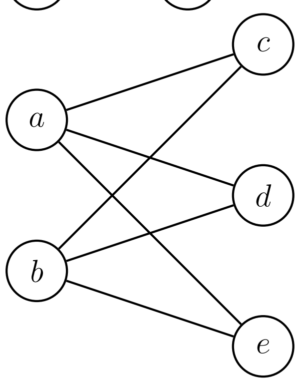
Its connectivity is 2 since removing the 2 vertices R and Q disconnects it, while removing any 1 vertex cannot disconnect it.

7. Consider the degree sequence $2, 2, 2, 3, 3$. Draw two **non-isomorphic** graphs which both have this degree sequence.

SOLUTION.

The following two graphs have degree sequence $2, 2, 2, 3, 3$, but are not isomorphic. To see they are not isomorphic, one way is to note that the first graph has a length 3 cycle, but the second graph does not.





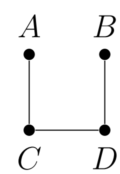
8. Consider the two proposed degree sequences. For each one state whether or not it is actually the degree sequence of a graph. If you say that it is the degree sequence of a graph, then draw that graph. If you say that it is **not** the degree sequence of any graph, prove this.

   (a) $1, 1, 2, 2$

   (b) $1, 3, 3, 4, 4$

SOLUTION.

1. The sequence $1, 1, 2, 2$ is a degree sequence of a graph. One such graph is:



2. The sequence $1, 3, 3, 4, 4$ is not a degree sequence of any graph.
To prove this, consider the sum of all degrees: $1 + 3 + 3 + 4 + 4 = 15$. For a graph, the sum of the degrees must be even, because it equals twice the number of edges. Here, the sum is odd, so this sequence cannot correspond to any graph.

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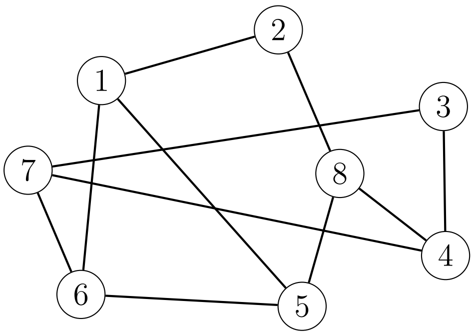
9. Consider the graph below.

   (a) Prove that it does **not** have an Eulerian Circuit.

   (b) Prove that it **does** have a Hamiltonian Circuit by finding one.



SOLUTION.

a) A graph has an Eulerian Circuit if and only if every vertex has an even degree. Since the degree of vetx 1 is 3, odd, there is no Eulerian circuit (could also have observed the oddness of vertex $4, 5, 6, 7$, or 8).

b) Next, to prove that the graph has a Hamiltonian Circuit, we need to find a path that visits each vertex exactly once and returns to the starting point. One possible Hamiltonian Circuit is:

$$1 \to 2 \to 8 \to 4 \to 3 \to 7 \to 6 \to 5 \to 1$$

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10. Suppose a graph has vertex set $\{1,2,3,4\}$ and adjacency matrix $B = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

You can trust that by a calculation $B^2 = \begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$

(a) How many length two walks are there from vertex 2 to vertex 4?

(b) How many length three walks are there from vertex 3 to vertex 1?

SOLUTION.

(a) The number of length two walks from vertex 2 to vertex 4 is given by the $(2,4)$ entry of B^2. Therefore, the number is 0.

(b) To find the number of length three walks from vertex 3 to vertex 1, we calculate the $(3,1)$ entry of the matrix $B^3 = B \cdot B^2$, which is the dot product $(1,1,0,0)\cdot(2,2,0,0) = 4$ Thus, the number of length three walks from vertex 3 to vertex 1 is 4.

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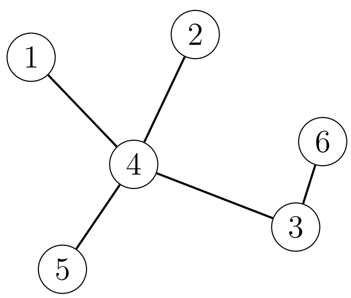
11. Suppose H is the graph pictured below. Define the following relation S on the vertices of the graph: $(u, v) \in S$ if the distance between u and v is at most 2.

(a) Write down all the pairs in relation S.

(b) Determine whether or not S is 1) reflexive, 2) irreflexive, 3) symmetric, 4) antisymmetric, 5) transitive.

(c) Is S an equivalence relation?



SOLUTION.

(a)
$$\begin{aligned} S = \{&(1,1),(1,2),(1,3),(1,4),(1,5),\\ &(2,1),(2,2),(2,3),(2,4),(2,5),\\ &(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\ &(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ &(5,1),(5,2),(5,3),(5,4),(5,5),\\ &(6,3),(6,4),(6,6)\}. \end{aligned}$$

(b) Determine properties of S:

1) Reflexive: For all $v \in V$, $(v, v) \in S$ since the distance from v to v is $0 \leq 2$.

2) Irreflexive: S is not irreflexive; for example $(1, 1)$ is in the relation.

3) Symmetric: True since distances are bidirectional.

4) Antisymmetric: No since we have $(1, 2)$ and $(2, 1)$ are in the relation.

5) Transitive: No. For example, $(1, 3)$ is in the relation since the distance is $2 \leq 2$, and also $(3, 6)$ is in the relation since their distance is $1 \leq 2$. But $(1, 6)$ is not in the relation, since their distance is $3 > 2$.

(c) S is an not an equivalence relation since it is not transitive.
