

SOLUTIONS
BRONX COMMUNITY COLLEGE
of the City University of New York
DEPARTMENT OF MATHEMATICS AND
COMPUTER SCIENCE

CSI 35

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YOUR NAME (first, then last):

Exam 2

SPRING 2026

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points (total scaled to 100).

1. Let h be defined by recursion as follows: $h(3) = 2$ and $h(n + 1) = 2h(n) - 3$. Write the definition of a Python function that takes one integer as input and returns the output of h on that input.
SOLUTION.

```
def h(n):  
    if n == 3:  
        return 2  
    else:  
        return 2 * h(n - 1) - 3
```

2. Consider the following binary relation on the set $\{A, B, C, D\}$. Determine whether or not it is 1) reflexive, 2) irreflexive, 3) symmetric, 4) antisymmetric, 5) transitive.

$$\{(A, A), (B, B), (C, C), (B, C), (B, D), (C, B), (D, B)\}$$

SOLUTION.

The given relation is $R = \{(A, A), (B, B), (C, C), (B, C), (B, D), (C, B), (D, B)\}$ on the set $S = \{A, B, C, D\}$.

1. ****Reflexive:**** A relation R is reflexive if for all $x \in S$, $(x, x) \in R$. In this case, (D, D) is missing.

The relation is not reflexive.

2. ****Irreflexive:**** A relation R is irreflexive if for all $x \in S$, $(x, x) \notin R$. Here, $(A, A), (B, B), (C, C) \in R$.

The relation is not irreflexive.

3. ****Symmetric:**** A relation R is symmetric if for all $(x, y) \in R$, $(y, x) \in R$. For example, $(B, C) \in R$ but $(C, B) \in R$; $(B, D) \in R$ but $(D, B) \in R$, so symmetry holds in the relevant pairs.

The relation is symmetric.

4. ****Antisymmetric:**** A relation R is antisymmetric if for all $(x, y) \in R$ and $(y, x) \in R$, $x = y$. We have $(B, C) \in R$ and $(C, B) \in R$ with $B \neq C$, violating antisymmetry.

The relation is not antisymmetric.

5. ****Transitive:**** A relation R is transitive if for all $(x, y) \in R$ and $(y, z) \in R$, $(x, z) \in R$. Consider $(B, C) \in R$ and $(C, B) \in R$, and $(B, B) \in R$, but does not cover cases like (B, D) and (D, B) to (B, B) .

The relation is not transitive.

3. Use induction to show that $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$, for $n \geq 1$.

SOLUTION.

Base case: For $n = 1$, the left-hand side is 1 and the right-hand side is

$$\frac{1(1+1)}{2} = \frac{2}{2} = 1.$$

So the statement is true for $n = 1$.

Induction hypothesis: Assume the statement is true for some $k \geq 1$, i.e.,

$$1 + 2 + \cdots + k = \frac{k(k+1)}{2}.$$

Inductive step: Show that the statement is true for $k + 1$, i.e.,

$$1 + 2 + \cdots + k + (k+1) = \frac{(k+1)(k+2)}{2}.$$

Starting from the induction hypothesis,

$$1 + 2 + \cdots + k + (k+1) = \frac{k(k+1)}{2} + (k+1).$$

Add $(k+1)$ to both sides:

$$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{k(k+1) + 2(k+1)}{2}.$$

Factor out $(k+1)$:

$$= \frac{(k+1)(k+2)}{2}.$$

Thus, the statement is also true for $k + 1$.

By induction, the formula $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$ holds for all $n \geq 1$.

4. Consider the set S defined by $14 \in S$ and $s + t \in S$ whenever $s \in S$ and $t \in S$. Show that every element of S is even.

SOLUTION.

To show that every element of S is even, we proceed by induction.

****Base Case:****

We include 14 in S . Clearly:

14 is even.

****Inductive Step:****

Assume $s \in S$ and $t \in S$ are even. Then:

$$s = 2k \quad \text{and} \quad t = 2m$$

for some integers k and m . Therefore:

$$s + t = 2k + 2m = 2(k + m)$$

Since $s + t$ is also of the form $2n$, where $n = k + m$, it follows that $s + t$ is even.

****Conclusion:****

By induction, every element of S must be even. \square

5. The set of binary strings is the set of all finite sequences of 0's and 1's (example: 011, 10100, etc). Define the following relation R on the binary strings.

$(x, y) \in R$ if strings x and y have the same number of 0s

For example: $(1001, 01011) \in R$, but $(010, 011) \notin R$. Prove that R is an equivalence relation.

SOLUTION.

To prove that R is an equivalence relation on the set of binary strings, we need to show that R is reflexive, symmetric, and transitive:

****Reflexive:**** For any binary string x , the number of 0s in x is equal to the number of 0s in x . Thus, $(x, x) \in R$.

****Symmetric:**** Suppose $(x, y) \in R$. This means the number of 0s in x is equal to the number of 0s in y . Therefore, the number of 0s in y is equal to the number of 0s in x , and thus $(y, x) \in R$.

****Transitive:**** Suppose $(x, y) \in R$ and $(y, z) \in R$. This means: - The number of 0s in x is equal to the number of 0s in y . - The number of 0s in y is equal to the number of 0s in z . Therefore, the number of 0s in x is equal to the number of 0s in z , so $(x, z) \in R$.

Since R is reflexive, symmetric, and transitive, it is an equivalence relation.

6. Let f be defined recursively by $f(1) = 2$, $f(2) = 6$, and $f(n) = f(n - 1) + f(n - 2)$.

(a) Evaluate $f(3)$ and $f(4)$.

(b) Prove that for $n \geq 1$, we have: $f(n)$ is even.

SOLUTION.

$$f(3) = f(2) + f(1) = 6 + 2 = 8$$

$$f(4) = f(3) + f(2) = 8 + 6 = 14$$

For the proof, we use induction.

****Base Case:**** For $n = 1$, $f(1) = 2$ which is even.

****Inductive Step:**** Assume $f(k)$ and $f(k - 1)$ are even for $k \geq 2$. We need to show $f(k + 1)$ is even.

Since $f(k) = f(k - 1) + f(k - 2)$, by the induction hypothesis, both $f(k - 1)$ and $f(k - 2)$ are even. Hence, $f(k)$ is even.

Then $f(k + 1) = f(k) + f(k - 1)$ is a sum of two even numbers, thus $f(k + 1)$ is even.

By induction, $f(n)$ is even for all $n \geq 1$.

7. Prove that every amount of postage of 6 cents or more can be formed using just 3-cent and 4-cent stamps. Use strong induction with the proof starting as follows:

- Let $P(n)$ be “postage of n cents can be formed using just 3-cent and 4-cent stamps.”
- Base cases: $P(6)$, $P(7)$, and $P(8)$. [You need to state and prove each base case]

Now finish the proof by doing the inductive step.

SOLUTION.

Base Cases:

$P(6)$:

A postage of 6 cents can be formed using two 3-cent stamps.

$$6 = 3 + 3$$

Therefore, $P(6)$ is true.

$P(7)$:

A postage of 7 cents can be formed using one 3-cent stamp and one 4-cent stamp.

$$7 = 3 + 4$$

Therefore, $P(7)$ is true.

$P(8)$:

A postage of 8 cents can be formed using two 4-cent stamps.

$$8 = 4 + 4$$

Therefore, $P(8)$ is true.

Inductive Step:

Assume for the inductive hypothesis that $P(k)$ is true for all k where $6 \leq k \leq n$, i.e., any amount of postage between 6 and n cents can be formed using 3-cent and 4-cent stamps.

We need to show that $P(n + 1)$ is true.

Consider $P(n + 1)$. Since $n \geq 8$, the value $n - 2 \geq 6$. By the inductive hypothesis, postage of $n - 2$ cents can be formed using 3-cent and 4-cent stamps. Thus, we can form postage of $n + 1$ cents by adding one 3-cent stamp to the $n - 2$ cents:

$$n + 1 = (n - 2) + 3$$

Since $n - 2$ can be formed with 3-cent and 4-cent stamps, $n + 1$ can also be formed using these stamps.

Therefore, by strong induction, every amount of postage of 6 cents or more can be formed using just 3-cent and 4-cent stamps.
