HW #4

Kerry Ojakian's CSI 35 Class

Due Date: Tuesday March 24 (beginning of class)

General Instructions: Homework must be stapled, be relatively neat, and have your name on it. All answers and work should be on this sheet. Use tutors, work with other students, but ... don't copy!

The Assignment

1. List the ordered pairs in the relation from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$ defined as follows.

   (a) $(a, b) \in R \Leftrightarrow gcd(a, b) = 1$

   (b) $(a, b) \in S \Leftrightarrow lcm(a, b) = 4$

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2. For each of these relations on the set $\{A, B, C, D, E\}$ decide whether it is reflexive, whether it is irreflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

   (a) $\{(B, B), (B, C), (B, D), (C, B), (C, C), (C, D)\}$

   (b) $\{(A, E), (E, A)\}$

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3. For each of these relations on the integers, decide whether it is reflexive, whether it is irreflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

   (a) $(x, y) \in R \Leftrightarrow xy \geq 2$

   (b) $(x, y) \in T \Leftrightarrow x \neq y$

4. Consider the relation on the positive integers defined by $(x, y) \in R$ if: x and y "have the same parity" (which means: both x and y are even, or both x and y are odd). Prove that R is an equivalence relation.

5. Consider the relation on the positive integers defined by xRy if: $x \leq y$. Show that R is NOT an equivalence relation.

6. The set of binary strings is the set of all finite sequences of 0's and 1's (example: 011, 10100, etc). Define the relation S on binary strings of length at least 4. $(x, y) \in S$ if: strings x and y are the same on their two rightmost bits. For example: $(10011, 011) \in S$, but $(0100, 0101) \notin S$. Is S an equivalence relation?

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7. Prove that for all positive integers n, n^3 is odd if and only if n is odd.

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8. Prove that $2^n > n^2$ if n is an integer greater than 4 (use induction).