

FIGURE 17 (a) The simple graphs G_1 and G_2 . (b) Their union $G_1 \cup G_2$.

Definition 9

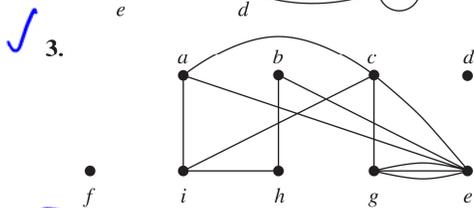
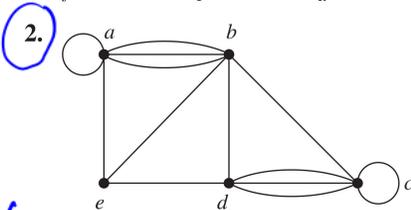
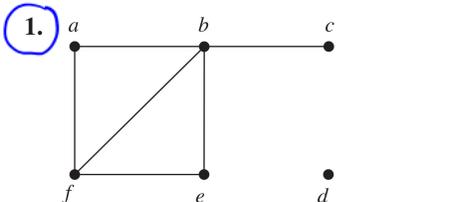
The *union* of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$. The union of G_1 and G_2 is denoted by $G_1 \cup G_2$.

EXAMPLE 20 Find the union of the graphs G_1 and G_2 shown in Figure 17(a). ◀

Solution: The vertex set of the union $G_1 \cup G_2$ is the union of the two vertex sets, namely, $\{a, b, c, d, e, f\}$. The edge set of the union is the union of the two edge sets. The union is displayed in Figure 17(b).

Exercises

In Exercises 1–3 find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.



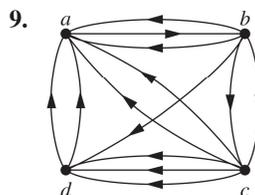
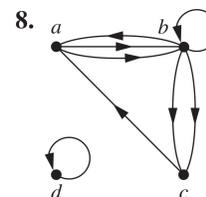
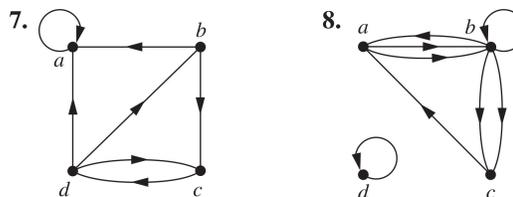
4. Find the sum of the degrees of the vertices of each graph in Exercises 1–3 and verify that it equals twice the number of edges in the graph.

for 1

5. Can a simple graph exist with 15 vertices each of degree five?

6. Show that the sum, over the set of people at a party, of the number of people a person has shaken hands with, is even. Assume that no one shakes his or her own hand.

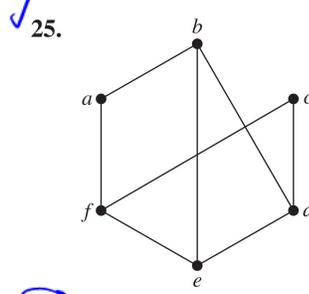
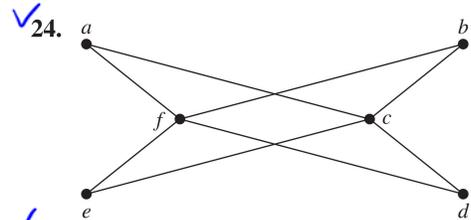
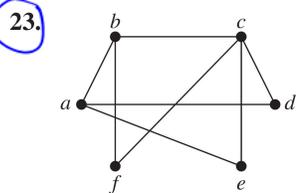
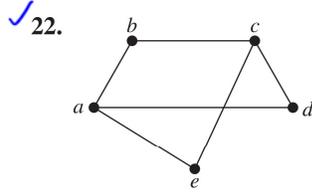
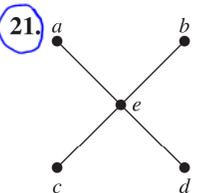
In Exercises 7–9 determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.



10. For each of the graphs in Exercises 7–9 determine the sum of the in-degrees of the vertices and the sum of the out-degrees of the vertices directly. Show that they are both equal to the number of edges in the graph.

11. Construct the underlying undirected graph for the graph with directed edges in Figure 2.
12. What does the degree of a vertex represent in the acquaintanceship graph, where vertices represent all the people in the world? What does the neighborhood of a vertex in this graph represent? What do isolated and pendant vertices in this graph represent? In one study it was estimated that the average degree of a vertex in this graph is 1000. What does this mean in terms of the model?
13. What does the degree of a vertex represent in an academic collaboration graph? What does the neighborhood of a vertex represent? What do isolated and pendant vertices represent?
14. What does the degree of a vertex in the Hollywood graph represent? What does the neighborhood of a vertex represent? What do the isolated and pendant vertices represent?
15. What do the in-degree and the out-degree of a vertex in a telephone call graph, as described in Example 4 of Section 10.1, represent? What does the degree of a vertex in the undirected version of this graph represent?
16. What do the in-degree and the out-degree of a vertex in the web graph, as described in Example 5 of Section 10.1, represent?
17. What do the in-degree and the out-degree of a vertex in a directed graph modeling a round-robin tournament represent?
18. Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.
19. Use Exercise 18 to show that in a group of people, there must be two people who are friends with the same number of other people in the group.
20. Draw these graphs.
 - a) K_7
 - b) $K_{1,8}$
 - c) $K_{4,4}$
 - d) C_7
 - e) W_7

In Exercises 21–25 determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.



26. For which values of n are these graphs bipartite?
 - a) K_n
 - b) C_n
 - c) W_n
 - d) Q_n
27. Suppose that there are four employees in the computer support group of the School of Engineering of a large university. Each employee will be assigned to support one of four different areas: hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware, networking, and wireless; Quiggley is qualified to support software and networking; Ruiz is qualified to support networking and wireless, and Sitea is qualified to support hardware and software.
 - a) Use a bipartite graph to model the four employees and their qualifications.
 - b) Use Hall's theorem to determine whether there is an assignment of employees to support areas so that each employee is assigned one area to support.
 - c) If an assignment of employees to support areas so that each employee is assigned to one support area exists, find one.
28. Suppose that a new company has five employees: Zamora, Agraharam, Smith, Chou, and Macintyre. Each employee will assume one of six responsibilities: planning, publicity, sales, marketing, development, and industry relations. Each employee is capable of doing one or more of these jobs: Zamora could do planning, sales, marketing, or industry relations; Agraharam could do planning or development; Smith could do publicity, sales, or industry relations; Chou could do planning, sales, or industry relations; and Macintyre could do planning, publicity, sales, or industry relations.
 - a) Model the capabilities of these employees using a bipartite graph.
 - b) Find an assignment of responsibilities such that each employee is assigned one responsibility.
 - c) Is the matching of responsibilities you found in part (b) a complete matching? Is it a maximum matching?
29. Suppose that there are five young women and five young men on an island. Each man is willing to marry some of the women on the island and each woman is willing to