SOLUTIONS

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

CSI 35 — Exam 1
Kerry Ojakian — FALL 2025
YOUR NAME (first, then last):

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points.

1. In Python, write the definition of a function F taking in an input (expected to be an integer). If the input is odd, then that same number is returned. If the input is even, then half of the number is returned.
   Example: $F(20)$ should return 10, while $F(25)$ should return 25

```
def F(n):
    if n % 2 == 0:
        return n/2
    else:
        return n
```

2. We recursively define f as follows: $f(1) = 2$ and $f(n) = 5 \cdot f(n-1) + 10$.
   Evaluate the following.

   (a) $f(1)$

   $= 2$

   (b) $f(2)$

   $= 5 \cdot f(1) + 10$
   $= 5 \cdot 2 + 10$
   $= \boxed{10}$

   (c) $f(3)$

   $= 5 \cdot f(2) + 10$
   $= 5 \cdot 10 + 10$
   $= \boxed{60}$

3. It seems that for $n \geq 4$ the following is true: $2^n < n!$ (recall that $n! = n \cdot (n-1) \cdots 2 \cdot 1$) Write a Python program that verifies that inequality for n such that $4 \leq n \leq 1000$.

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for n in range(4,1001):
    if 2^n >= n!:
        print("False")
```

Issue! Defing 2^n and n! in python. Sorry!

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4. Let $F(n) = -3n$, defined on integers $n \geq 1$. Find a recursive definition for F.

$F(1) = -3$

$F(n) = -3 + F(n-1)$

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5. Consider the following program:

if $x < 0$:
  $x = x + 7$
else:
  $x = 6x$

Suppose you are given that x is initialized to some positive integer. Show that after running the program, the value of x is even.

1. Since x is positive $x<0$ is False
2. Thus x is set to $6x = 2(3x)$
3. So value of x is $2 \cdot k$ for some integer k
4. Thus x is even

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6. Prove that for all positive integers n, n^2 is odd if and only if n is odd.

I ($\Rightarrow$)

1. Assume for contradiction: n even
2. Thus $n = 2k$, some integer k
3. Thus $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$
4. Thus n^2 is even
5. n^2 is even and odd. Contradiction

II ($\Leftarrow$)

1. Given n is odd
2. Thus $n = 2k+1$, some int k
3. $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2+2k)+1$
4. Thus n^2 is odd

7. Consider the set S which is defined recursively as follows:

$2 \in S$ and $5 \in S$

If $x \in S$ and $y \in S$, then $x + y \in S$.

(a) Is 20 in S? Is 13 in S?

(b) Find all the positive integers that are **not** in S.

a) $20 = 5+5+5+5$ so yes
$13 = 5+2+2+2+2$ so yes

b) 1 and 3. Since $\overset{2+2}{4}$, 5 in S, then adding 2 to 4 gets larger even, and adding 2 to 5, larger odd.

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8. Consider the proposition: 5 divides $n^5 - n$ for $n \geq 2$.
In order to prove this by induction you should 1) state what statement $P(n)$ you are doing induction on, 2) show $P(n)$ is true for the base case, and 3) do the inductive step.
Just do (1) and (2). That is, state exactly what $P(n)$ is, then state and prove the base case.
Do **not** do the inductive step.

- $P(n)$: " 5 divides $n^5 - n$ "
- Base case: $P(2)$: " 5 divides $2^5 - 2$ "
  which is: $5 \mid 30$, true since $30 = 5 \cdot (6)$

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9. Prove that $2 + 4 + 6 + \cdots + 2n = n(n + 1)$
(i.e. the sum of the first n positive even integers is $n(n + 1)$. You must use induction to prove this.

1 Let $P(n)$ be "$2 + \dots + 2n = n(n+1)$"
2 Base Case $P(1)$: "$2 = 1(1+1)$" ← $2=2$, true.
3 $P(k)$: $2 + \dots + 2k = k(k+1)$
4 Add $2(k+1)$ both sides $P(k)$ equation:
$2 + \dots + 2k + 2(k+1) = k(k+1) + 2(k+1)$
5 Factor $(k+1)$ on right:
$2 + \dots + 2(k+1) = (k+1)(k+2)$
which is $P(k+1)$
6 Thus showed inductive step of $P(k) \Rightarrow P(k+1)$

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10. Prove that every amount of postage of 6 cents or more can be formed using just 3-cent and 4-cent stamps. Use strong induction with the proof starting as follows:

   - Let $P(n)$ be "postage of n cents can be formed using just 3-cent and 4-cent stamps."
   - Base cases: $P(6), P(7)$, and $P(8)$. [You need to state and prove each base case]

   Now finish the proof by doing the inductive step.

$P(6)$: "6 cents with 3-cent, 4-cent" : $6 = 3+3$, true

$P(7)$: "7 cents with 3-cent, 4-cent" : $7 = 3+4$, true

$P(8)$: "8 cent with 3-cent, 4-cent" : $8 = 4+4$, true

Inductive step: Show $P(k)$: "k cents with 3, 4-cent"
$(k \geq 9)$. By $P(k-3)$ I.H. can do $k-3 = a3 + b4$
some a, b. [note $k-3 \geq 6$]

Thus $(a+1)3 + b4$ (i.e. one more 3-cent stamp)
equals $k-3+3 = k$
so $P(k)$ shown.