

CSI 35 LECTURE NOTES (Ojakian)
Topic 12: Coloring and Matchings in Graphs

OUTLINE

(References: Wells: 157, 159, Rosen: 10.2, 10.8)

1. Coloring

(a) Motivation: Scheduling Problem

Goal: Schedule a number of student exams so no student has a time conflict.

- i. Make a graph where each vertex is an exam
- ii. Put an edge between vertices if some student has both exams
- iii. So what is a proper coloring?

(b) Chromatic Number

Typical goal: Use as few colors as needed.

PROBLEM 1. *Find the chromatic number of the following: K_n and C_n and W_n ($n + 1$ vertices) and Trees*

PROBLEM 2. *Wells 159.2.2 (coloring bipartite)*

(c) Exercises

Section 10.8: 5 - 11 (Finding chromatic number)

Section 10.8: 12 - 13, 16 - 20

2. Matching (10.2)

- (a) One Motivation: Match Jobs to workers (jobs are side that needs to be covered, can miss some workers).

PROBLEM 3. *Create problem from students in class, along with jobs to perform. Is there a complete matching from students to jobs?
(or Example 14 (p. 692))*

- (b) Setup.

- i. Have graph with bipartition (V_1, V_2) in a graph.
- ii. Considering just matchings (i.e - no edges can touch)
- iii. Issue: Can you cover V_1 or not? (pick one of side to cover)

- (c) Old fashioned marriage example (between men and women)

- (d) Try to find Necessary conditions for having such a matching (i.e. If there is a a matching that covers V_1 , what must be true).

- (e) Statement Hall's Marriage Theorem. Prove necessity of condition; leave other direction open

- (f) Examples

- i.

PROBLEM 4. *Consider the graph on the board (with 3 vertices on the left). Does it satisfy the condition in Hall's Theorem? If not add, some edges so that it does satisfy the condition.*

- ii. Do more.

- (g) Exercises (10.2)

- i. Exercises: 27 - 30
- ii. Exercises (hard??): 31 - 33