# CSI 35 LECTURE NOTES (Ojakian)

## Topic 11: Walks in Graphs: Eulerian, Hamiltonian, and Counting

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**OUTLINE**
(References: Wells: 157, 159, Rosen: 10.4, 10.5)

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1. Traversing an entire graph
   (a) Why would you want to traverse an entire graph?
   (b) Any requirements and/or goals of such a traversal.
2. Ways to step through graph
   For each, we have a start vertex and an end vertex (maybe start = end)
   (a) Walk: any repeats - of vertices and edges
   (b) Trail: no edge repeats, any amount of vertex repeats
   (c) Path: no repeats (even start and end may not repeat)
   (d) Shortest Path (and distance and diameter)
3. Eulerian
   (a) Suppose the vertices are intersections and the edges are streets. Your goal is to clean all the streets returning to your start. Can you do it without repeating a street?

   **PROBLEM 1.** *Wells exercise 157.2.1 (Eulerian)*

   (b) Concept of "Necessary and Sufficient Conditions"
      i. Example: Integer divisible by 3 and 4.
   (c) Eulerian: Necessary and Sufficient Conditions?
      i. Try finding some necessary ones? some sufficient ones? both necessary and sufficient??
      ii. Statement of Euler's Theorem
   (d) Proof of Euler's Theorem (via "iterations")

   **PROBLEM 2.** *For an Eulerian graph, find an Eulerian Circuit using the method from the proof of Euler's Theorem. Do it so that in each iteration, a circuit is chosen that does not repeat vertices (and even better, try to make it the shortest circuit option at that point).*

4. Hamiltonian

   (a) Suppose there is a graph in which the vertices represent cities and the edges are roads. You are a salesperson and want to visit every city. Can you do it without repeating cities?

       **PROBLEM 3.** *Wells exercise 157.3.3 (Hamiltonian)*

   (b) Hamiltonian: Necessary and Sufficient Conditions?

       i. Try finding some necessary ones? some sufficient ones? both necessary and sufficient??
       ii. Statement of Dirac's Theorem (note: converse false - Exercise 48)

   (c) Applications

       i. Gray codes and the hypercube

5. Comparing Eulerian and Hamiltonian

   **PROBLEM 4.** *Wells exercise 157.4.2 and 157.4.3 (which Eulerian? which Hamiltonian?)*

6. Counting the walks (section 10.4)

   (a) How many walks between two vertices: vertex disjoint and not?

       i. Matrix product (just for square matrices)
       ii. Matrix product theorem FOR counting number of walks

           **PROBLEM 5.** *For a small graph, finds its adjacency matrix, and various powers. Compare this to the vertex disjoint walks and the corresponding vertex connectivity.*

7. Exercises

   (a) Section 10.5. Euler. 1 - 8 (ignore "path" part), 10 (graph corresponding graph)
   (b) Section 10.5. Euler. 26, 28a.
   (c) Section 10.5. Ham. 30 - 36, 44, 45, 47 (skip "Ore" part ii ), 55
   (d) Section 10.4: 19, 24, 25