CSI 35 LECTURE NOTES (Ojakian)

Topic 5: Inductive Proofs about Recursion

OUTLINE

(References: Wells sections 105-107, 124, 125; Rosen: 5.3, 5.4)

1. Inductive Proofs about Recursion

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1. <u>Inductive Proofs about recursively defined functions</u>

   (a) Two views.

      i. Proving a recursive function has a certain property

      ii. Proving a recursive programs works.

   (b) For $F(0) = 1$; $F(n) = 2 \cdot F(n-1)$: Prove that $F(n) = 2^n$

   (c) Example of finding and solving (Wells 124.2.3): Compounding interest annually.

      i. Let $A(0) = P$ (the principal)

      ii. $A(n+1) = \cdots$

   (d) Do Wells exercise 107.3.12 (page 162); using total induction.

   (e) Prove C(n,k) formula (Thm 125.6) by induction (probably skip!)

2. <u>Structural Inductive Proofs</u>

   (a) Form.

      i. Prove the property is true of all elements specified in the basis step.

      ii. Prove that the recursive step preserves the property (i.e. if the property holds of given elements, then it holds of an element constructed from these)

   (b) Recall divisibility rule: If $a|x$ and $a|y$, then $a|(x+y)$.

   (c) Section 5.3, Example 10 (about set with 3 and closed under +), but just show: Every element of S is divisible by 3.

   (d) Section 5.3, Exercise 26. Then they do 27.

   (e)

   PROBLEM 1. *Define a subset of binary strings S as follows (implicit operation is concatenation):* $01 \in S$ *and* $10 \in S$ *and if* $x \in S$ *then* $xy \in S$.

      i. List all the length 4 and length 5 binary strings in S

      ii. Prove that the strings in S all have an equal number of 0's and 1's.