CSI 35 LECTURE NOTES (Ojakian)

Topic 2: Mathematical Induction

OUTLINE

(References: Wells 102-104, Rosen 5.1)

1. Mathematical Induction

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1. Summation Notation

   (a) Compute some examples.

   PROBLEM 1. *Use the summation notation to write the sum of the positive even integers from 2 up to and including 1000.*

2. First Induction Proof

   PROBLEM 2. *Prove that* $1 + 2 + \ldots + n = n(n + 1)/2$ *(example 1)*

   (a) Base Step

   (b) Inductive Step

   (c) Some pictures of it

      i. Dominos

      ii. The infinitely long subway route (exercise 1; show that the subway stops at all the stations).

      iii. Ascending a ladder

   (d) Cheap ones to practice structure

   PROBLEM 3. *Prove that* $1 + 1 + \ldots + 1$ *(*n *times)* $= n$ *by induction!*

   PROBLEM 4. *Prove that* $2 + 2 + \ldots + 2$ *(*n *times)* $< 5n$ *by induction!*

3. More Inductive Proofs

   (a) Do one: $n < 2^n$ (example 5) or $2^n < n!$ (example 6)

   (b) Prove that $n^3 - n$ is divisible by 3, when n is a positive integer (example 8).

   (c) Prove that the number of subsets of $[n]$ is 2^n (example 10).

   (d) (Probably skip) Let n be a positive integer. Show tht every 2^n by 2^n checkerboard with one square removed (anywhere) can be tiled using "right triominoes." (example 14)

4. Mistaken Inductive Proofs

   (a) Example 15

   (b) Exercises: 49, 50, 51

5. Exercises (Section 5.1)

(a) Equalities: 2 - 5, 11

(b) Inequalities: 18 - 23

(c) Divisibility: 31 - 34

(d) Structural: 45, 46

    PROBLEM 5. *Suppose that* n *is a positive integer and* f *is a function from* $[n+1]$ *to* $[n]$*. Use mathematical induction to show that* f *is* **not** *one-to-one.*

(e) Miscellaneous: 56, 57 (good calculus one!), 60