HW #5

Kerry Ojakian's CSI 35 Class
Due Date: Thursday November 13 (beginning of class)

General Instructions:

- Homework must be stapled, be relatively neat, and have your name on it.
- Use tutors, work with other students, but ... don't copy!

The Assignment

1. Consider the the graph on vertices $\{1, 2, 3, 4, 5, 6\}$ with edges:

   $$\{1, 2\}, \{2, 3\}, \{1, 3\}, \{4, 5\}\{5, 6\}$$

   Draw it, and write down its adjacency matrix.

2. Consider the following adjacency matrix for a graph on vertex set $\{1, 2, 3, 4, 5\}$.

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Draw the graph, write down its edge set, and find its adjacency list representation. What is the common name for this graph? (i.e. from among the classes P_n, C_n, K_n, $K_{a,b}$, etc)

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3. Draw K_4. Find a connected subgraph which is not an induced subgraph. Find a different graph from K_4 such that every connected subgraph is in fact an induced subgraph (choose a small example! and verify by checking all connected subgraphs).

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4. How many edges do each of the following graphs have? (n, a, b are all positive integers)

$$P_n \quad C_n \quad K_{a,b} \quad K_n$$

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5. Consider C_4 expressed as follows: Its vertex set is $\{1, 2, 3, 4\}$; its edge set is $\{\ \{1, 2\},\ \{2, 3\},\ \{3, 4\},\ \{4, 1\}\ \}$.
   Find all the graphs (on the same vertex set) which are not the same, but are isomorphic.

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6. Consider P_4 expressed as follows: Its vertex set is $\{1, 2, 3, 4\}$; its edge set is $\{\ \{1, 2\},\ \{2, 3\},\ \{3, 4\},\ \}$.
   Find all the graphs (on the same vertex set) which are not the same, but are isomorphic.

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7. Find two graphs with the same degree sequence which are not isomorphic (don't look it up!)

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8. Draw a strongly connected directed graph with 4 arcs and 4 vertices. Write down its adjacency matrix.

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9. Suppose the degree sequence of a graph is $1, 2, 2, 3, 3, 3$. How many edges are in the graph?

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10. Suppose someone tells you they have a graph with degree sequence $1, 2, 2, 2, 4, 5, 5$. How do you know they are lying? (or at least mistaken ...)

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11. Consider the graph on vertex set $\{A, B, C, D\}$, with edges $\{A, B\}, \{A, C\}, \{A, D\}$. Draw all its subgraphs which have at least one edge.

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12. Consider the graph on vertex set $\{A, B, C, D\}$, with edges

$$\{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, D\}.$$

Draw all its *induced* subgraphs which have at least one edge.

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13. Consider the graph given by the following adjacency list representation.

A	B, C
B	A, C, D
C	A, B, D
D	B, C

Draw the graph. Find its adjacency matrix. What is its connectivity?