HW #4

Kerry Ojakian's CSI 35 Class
Due Date: Tuesday October 28 (beginning of class)

General Instructions: Homework must be stapled, be relatively neat, and have your name on it. All answers and work should be on this sheet. Use tutors, work with other students, but ... don't copy!

The Assignment

1. Consider the relation on the positive integers defined by $(x, y) \in R$ if: x and y "have the same parity" (which means: both x and y are even, or both x and y are odd). Prove that R is an equivalence relation.

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2. Consider the relation on the positive integers defined by xRy if: $x \leq y$. Show that R is NOT an equivalence relation.

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3. The set of binary strings is the set of all finite sequences of 0's and 1's (example: 011, 10100, etc). Define the relation S on binary strings of length at least 4. $(x, y) \in S$ if: strings x and y are the same on their two rightmost bits. For example: $(10011, 011) \in S$, but $(0100, 0101) \notin S$. Is S an equivalence relation?

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4. Which of these relations on $\{A, B, C, D\}$ are partial orderings? If a relation is not a partial ordering, determine which relevant properies it lacks.

   (a) $\{(A, B), (B, C), (C, D), (A, C), (A, D), (B, D), (A, A), (B, B), (C, C)\}$

   (b) $\{(A, A), (B, B), (B, A), (C, C), (A, B)\}$

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5. Draw the Hasse diagram for the in inclusion relation on the power set of $\{s, t, v, w\}$

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6. Draw the Hasse diagram for the divisibility poset for each set of integers (i.e. (a, b) is in the poset when $a|b$). Indicate any least or greatest elements.

   (a) $\{1, 2, 3, 4, 6, 8, 24\}$

   (b) $\{3, 6, 9, 12, 15, 18, 36\}$

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7. Prove that for all positive integers n, n^3 is odd if and only if n is odd.

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8. Prove that $2^n > n^2$ if n is an integer greater than 4 (use induction).
