HW #3

Kerry Ojakian's CSI 35 Class
Due Date: Thursday October 16 (beginning of class)

General Instructions:

- Homework must be stapled, be relatively neat, and have your name on it. All work and answers should be on this sheet.
- Use tutors, work with other students, but ... don't copy!

The Assignment

1. We recursively define g as follows: $g(1) = 1$ and $g(n+1) = (g(n))^2 + g(n)$. Evaluate the following

   (a) $g(2)$

   (b) $g(3)$

   (c) $g(4)$

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2. We recursively define f as follows: $f(0) = 1$, $f(1) = 1$ and $f(n) = f(n-1) - f(n-2)$. Evaluate the following.

   (a) $f(1)$

   (b) $f(3)$

   (c) $f(5)$

   Can you describe f without recursion?

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3. Let $g(n) = (-3)^n$, defined on integers $n \geq 1$. Find a recursive definition for g.

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4. Let g be defined by recursion as follows: $g(2) = 3$ and $g(n) = 7g(n-1) - 2$. Write the definition of a Python function that takes one integer as input and returns the output of g on that input.

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5. Consider the set S defined by $14 \in S$ and $s + t \in S$ whenever $s \in S$ and $t \in S$. Show that every element of S is even.

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6. Let r be defined recursively by $r(1) = 3$, $r(2) = 5$ and $r(n+1) = r(n) + 4 \cdot r(n-1)$. Prove that $r(n)$, for $n \geq 1$ is always odd.

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7. List the ordered pairs in the relation from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$ defined as follows.

   (a) $(a, b) \in R \Leftrightarrow gcd(a, b) = 1$

   (b) $(a, b) \in S \Leftrightarrow lcm(a, b) = 4$

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8. For each of these relations on the set $\{A, B, C, D, E\}$ decide whether it is reflexive, whether it is irreflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

   (a) $\{(B, B), (B, C), (B, D), (C, B), (C, C), (C, D)\}$

   (b) $\{(A, E), (E, A)\}$

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9. For each of these relations on the integers, decide whether it is reflexive, whether it is irreflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

   (a) $(x, y) \in R \Leftrightarrow xy \geq 2$

   (b) $(x, y) \in T \Leftrightarrow x \neq y$
