HW #1

Kerry Ojakian's CSI 35 Class

Due Date: Thursday September 4 (beginning of class)

General Instructions:

- Homework must be stapled, be relatively neat, and have your name on it.
- Use tutors, work with other students, but ... don't copy!

The Assignment

1. Read the first 3 sections of the Krantz proof article (at webpage next to Topic 1). Write a brief summary of it (a page). Write about one point you agree with and one point you disagree with, and why. Do **not** use any AI assistance. Attach your typed response.

2. Suppose variables x and y already have values assigned to them. Write code that swaps the values between x and y (using an extra variable besides x and y).

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3. Define a function which takes one numerical input. If the input is negative, it returns 0; otherwise it just returns back the same input. Graph this function.
   Note: This function is famous in the AI neural net work; called the "ReLU" function.

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4. Suppose L is a list of positive integers, and b is a boolean variable. Write code that sets b to True if every entry in the list is even, and False otherwise.

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5. Some day we will prove that for all positive integers n, $1 + 2 + \cdots + n = \dfrac{n(n+1)}{2}$. Write a program that verifies this claim for n such that $1 \leq n \leq 1000$.

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6. Write a program that finds the sum of the first n odd positive integers; for example, if $n = 2$, the program should calculate 4, since $1+3 = 4$. Make conjecture about the value of this sum (no need to prove it! however, use data from your programs as evidence of your conjecture).

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7. Prove that the sum of any three odd integers is odd.

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8. Prove or disprove: If $a|bc$ then $a|b$ or $a|c$.

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9. Consider the following program:

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\begin{array}{l}
\text{if } x > 9: \\
\qquad x = 9 \\
\text{elif } x < -9: \\
\qquad x = -9 \\
\text{else:} \\
\qquad x = 0
\end{array}
$$

   (a) Suppose the initial assertion is that $x = -70$. Determine what the final value of x will be, and prove it.

   (b) What are all the possible final values of x? Prove it (Hint: Use cases)

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10. Suppose n is an integer. Prove n is odd if and only if $n + 101$ is even.

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11. We know that the product of 2 consecutive positive integers is divisible by 2, and that the product of any 3 consecutive positive integers is divisible by 3.
Make a conjecture about k integers. Prove it!

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12. Prove that a right triangle *cannot* have all its side lengths equal to prime numbers (Hint: use Pythagorean Theorem and proof by contradiction).

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13. A **perfect square** is an integer that is equal to the square of another integer; for example 9 is a perfect square because $9 = 3^2$, but 18 is *not* a perfect square.

    (a) Prove that if m and n are both perfect squares, then nm is also a perfect square.

    (b) Prove or disprove: The sum of two perfect squares is a perfect square.
