CLASS WORK #17

6. Can the leaves of an ordered rooted tree have the following list of universal addresses? If so, construct such an ordered rooted tree.

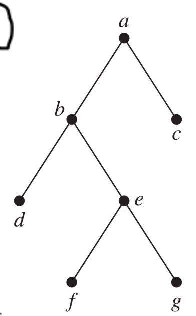
a) 1.1.1, 1.1.2, 1.2, 2.1.1.1, 2.1.2, 2.1.3, 2.2, 3.1.1, 3.1.2.1, 3.1.2.2, 3.2

b) 1.1, 1.2.1, 1.2.2, 1.2.3, 2.1, 2.2.1, 2.3.1, 2.3.2, 2.4.2.1, 2.4.2.2, 3.1, 3.2.1, 3.2.2

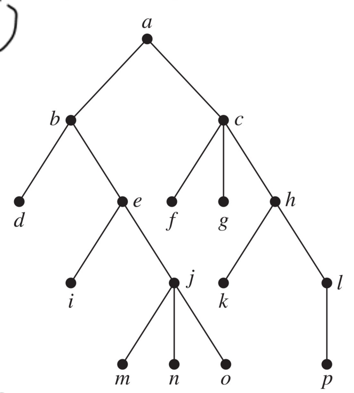
c) 1.1, 1.2.1, 1.2.2, 1.2.2.1, 1.3, 1.4, 2, 3.1, 3.2, 4.1.1.1

In Exercises 7–9 determine the order in which a preorder traversal visits the vertices of the given ordered rooted tree.

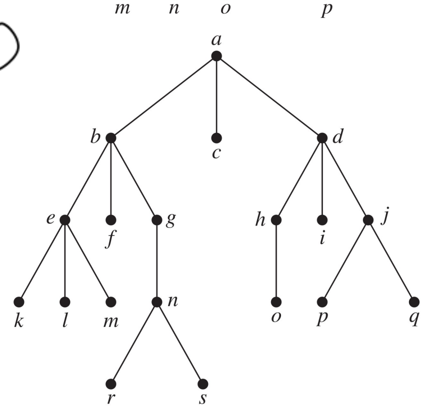
c **7.**



c **8.**



c **9.**



c

10. In which order are the vertices of the ordered rooted tree in Exercise 7 visited using an inorder traversal?

11. In which order are the vertices of the ordered rooted tree in Exercise 8 visited using an inorder traversal?

12. In which order are the vertices of the ordered rooted tree in Exercise 9 visited using an inorder traversal?

13. In which order are the vertices of the ordered rooted tree in Exercise 7 visited using a postorder traversal?

14. In which order are the vertices of the ordered rooted tree in Exercise 8 visited using a postorder traversal?

c **15.** In which order are the vertices of the ordered rooted tree in Exercise 9 visited using a postorder traversal?

b **16.** **a)** Represent the expression $((x+2) \uparrow 3) * (y-(3+x)) - 5$ using a binary tree.

Write this expression in

b) prefix notation.

c) postfix notation.

d) infix notation.

b **17.** **a)** Represent the expressions $(x + xy) + (x/y)$ and $x + ((xy + x)/y)$ using binary trees.

Write these expressions in

b) prefix notation.

c) postfix notation.

d) infix notation.

b **18.** **a)** Represent the compound propositions $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$ and $(\neg p \wedge (q \leftrightarrow \neg p)) \vee \neg q$ using ordered rooted trees.

Write these expressions in

b) prefix notation.

c) postfix notation.

d) infix notation.

b **19.** **a)** Represent $(A \cap B) - (A \cup (B - A))$ using an ordered rooted tree.

Write this expression in

b) prefix notation.

c) postfix notation.

d) infix notation.

D

***20.** In how many ways can the string $\neg p \wedge q \leftrightarrow \neg p \vee \neg q$ be fully parenthesized to yield an infix expression?

***21.** In how many ways can the string $A \cap B - A \cap B - A$ be fully parenthesized to yield an infix expression?

b **22.** Draw the ordered rooted tree corresponding to each of these arithmetic expressions written in prefix notation. Then write each expression using infix notation.

a) $+ * + - 5\ 3\ 2\ 1\ 4$

b) $\uparrow + 2\ 3 - 5\ 1$

c) $* / 9\ 3 + * 2\ 4 - 7\ 6$

A **23.** What is the value of each of these prefix expressions?

a) $- * 2 / 8\ 4\ 3$

b) $\uparrow - * 3\ 3 * 4\ 2\ 5$

c) $+ - \uparrow 3\ 2 \uparrow 2\ 3 / 6 - 4\ 2$

d) $* + 3 + 3 \uparrow 3 + 3\ 3\ 3$

A **24.** What is the value of each of these postfix expressions?

a) $5\ 2\ 1 - - 3\ 1\ 4 + + *$

b) $9\ 3 / 5 + 7\ 2 - *$

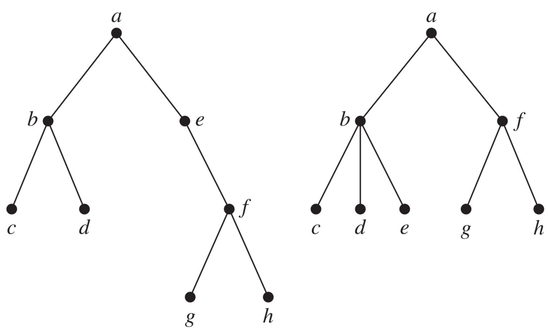
c) $3\ 2 * 2 \uparrow 5\ 3 - 8\ 4 / * -$

25. Construct the ordered rooted tree whose preorder traversal is *a, b, f, c, g, h, i, d, e, j, k, l*, where *a* has four children, *c* has three children, *j* has two children, *b* and *e* have one child each, and all other vertices are leaves.

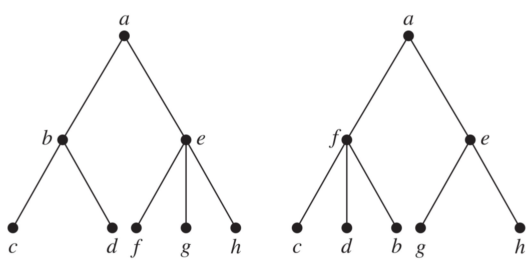
***26.** Show that an ordered rooted tree is uniquely determined when a list of vertices generated by a preorder traversal of the tree and the number of children of each vertex are specified.

***27.** Show that an ordered rooted tree is uniquely determined when a list of vertices generated by a postorder traversal of the tree and the number of children of each vertex are specified.

C **28.** Show that preorder traversals of the two ordered rooted trees displayed below produce the same list of vertices. Note that this does not contradict the statement in Exercise 26, because the numbers of children of internal vertices in the two ordered rooted trees differ.



C **29.** Show that postorder traversals of these two ordered rooted trees produce the same list of vertices. Note that this does not contradict the statement in Exercise 27, because the numbers of children of internal vertices in the two ordered rooted trees differ.



Well-formed formulae in prefix notation over a set of symbols and a set of binary operators are defined recursively by these rules:

(*i*) if x is a symbol, then x is a well-formed formula in prefix notation;

(*ii*) if X and Y are well-formed formulae and $*$ is an operator, then $*\,XY$ is a well-formed formula.

30. Which of these are well-formed formulae over the symbols $\{x, y, z\}$ and the set of binary operators $\{\times, +, \circ\}$?

a) $\times + + x\,y\,x$

b) $\circ\, x\,y \times x\,z$

c) $\times \circ x\,z \times \times x\,y$

d) $\times + \circ\, x\,x \circ x\,x\,x$

***31.** Show that any well-formed formula in prefix notation over a set of symbols and a set of binary operators contains exactly one more symbol than the number of operators.

32. Give a definition of well-formed formulae in postfix notation over a set of symbols and a set of binary operators.

33. Give six examples of well-formed formulae with three or more operators in postfix notation over the set of symbols $\{x, y, z\}$ and the set of operators $\{+, \times, \circ\}$.

34. Extend the definition of well-formed formulae in prefix notation to sets of symbols and operators where the operators may not be binary.

11.4 Spanning Trees

11.4.1 Introduction

Consider the system of roads in Maine represented by the simple graph shown in Figure 1(a). The only way the roads can be kept open in the winter is by frequently plowing them. The highway department wants to plow the fewest roads so that there will always be cleared roads connecting any two towns. How can this be done?

At least five roads must be plowed to ensure that there is a path between any two towns. Figure 1(b) shows one such set of roads. Note that the subgraph representing these roads is a tree, because it is connected and contains six vertices and five edges.

This problem was solved with a connected subgraph with the minimum number of edges containing all vertices of the original simple graph. Such a graph must be a tree.

Definition 1 Let G be a simple graph. A *spanning tree* of G is a subgraph of G that is a tree containing every vertex of G.