s_0 goes just to s_1 and s_2 goes just to s_4 on input of 1 in the nondeterministic machine; and the set $\{s_1, s_4\}$ goes to $\{s_3\}$ on input of 0, because s_1 and s_4 both go to just s_3 on input of 0 in the deterministic machine. All subsets that are obtained in this way are included in the deterministic finite-state machine. Note that the empty set is one of the states of this machine, because it is the subset containing all the next states of $\{s_3\}$ on input of 1. The start state is $\{s_0\}$, and the set of final states are all those that include s_0 or s_4. ◀

Exercises

1. Let $A = \{0, 11\}$ and $B = \{00, 01\}$. Find each of these sets.

a) AB **b)** BA **c)** A^2 **d)** B^3

2. Show that if A is a set of strings, then $A\emptyset = \emptyset A = \emptyset$.

3. Find all pairs of sets of strings A and B for which $AB = \{10, 111, 1010, 1000, 10111, 101000\}$.

4. Show that these equalities hold.

a) $\{\lambda\}^* = \{\lambda\}$

b) $(A^*)^* = A^*$ for every set of strings A

5. Describe the elements of the set A^* for these values of A.

a) $\{10\}$ **b)** $\{111\}$ **c)** $\{0, 01\}$ **d)** $\{1, 101\}$

6. Let V be an alphabet, and let A and B be subsets of V^*. Show that $|AB| \le |A||B|$.

7. Let V be an alphabet, and let A and B be subsets of V^* with $A \subseteq B$. Show that $A^* \subseteq B^*$.

8. Suppose that A is a subset of V^*, where V is an alphabet. Prove or disprove each of these statements.

a) $A \subseteq A^2$ **b)** if $A = A^2$, then $\lambda \in A$

c) $A\{\lambda\} = A$ **d)** $(A^*)^* = A^*$

e) $A^*A = A^*$ **f)** $|A^n| = |A|^n$

9. Determine whether the string 11101 is in each of these sets.

a) $\{0, 1\}^*$ **b)** $\{1\}^*\{0\}^*\{1\}^*$

c) $\{11\}\{0\}^*\{01\}$ **d)** $\{11\}^*\{01\}^*$

e) $\{111\}^*\{0\}^*\{1\}$ **f)** $\{11, 0\}\{00, 101\}$

10. Determine whether the string 01001 is in each of these sets.

a) $\{0, 1\}^*$ **b)** $\{0\}^*\{10\}\{1\}^*$

c) $\{010\}^*\{0\}^*\{1\}$ **d)** $\{010, 011\}\{00, 01\}$

e) $\{00\}\{0\}^*\{01\}$ **f)** $\{01\}^*\{01\}^*$

11. Determine whether each of these strings is recognized by the deterministic finite-state automaton in Figure 1.

a) 111 **b)** 0011 **c)** 1010111 **d)** 011011011

12. Determine whether each of these strings is recognized by the deterministic finite-state automaton in Figure 1.

a) 010 **b)** 1101 **c)** 1111110 **d)** 010101010

13. Determine whether all the strings in each of these sets are recognized by the deterministic finite-state automaton in Figure 1.

a) $\{0\}^*$ **b)** $\{0\}\{0\}^*$ **c)** $\{1\}\{0\}^*$

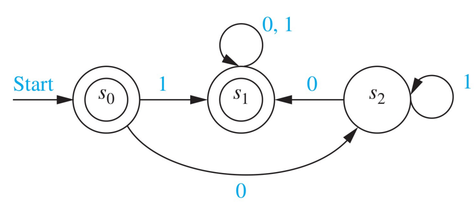
d) $\{01\}^*$ **e)** $\{0\}^*\{1\}^*$ **f)** $\{1\}\{0, 1\}^*$

14. Show that if $M = (S, I, f, s_0, F)$ is a deterministic finite-state automaton and $f(s, x) = s$ for the state $s \in S$ and the input string $x \in I^*$, then $f(s, x^n) = s$ for every nonnegative integer n. (Here x^n is the concatenation of n copies of the string x, defined recursively in Exercise 37 in Section 5.3.)

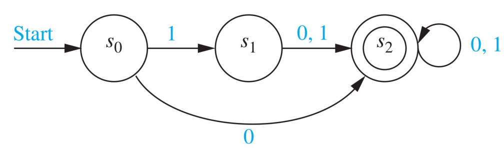
15. Given a deterministic finite-state automaton $M = (S, I, f, s_0, F)$, use structural induction and the recursive definition of the extended transition function f to prove that $f(s, xy) = f(f(s, x), y)$ for all states $s \in S$ and all strings $x \in I^*$ and $y \in I^*$.

In Exercises 16–22 find the language recognized by the given deterministic finite-state automaton.

16.



17.



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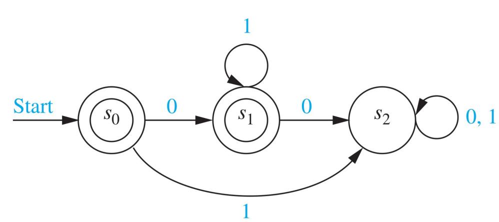

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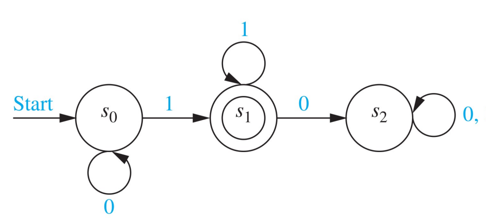

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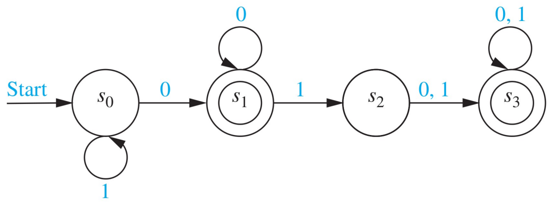

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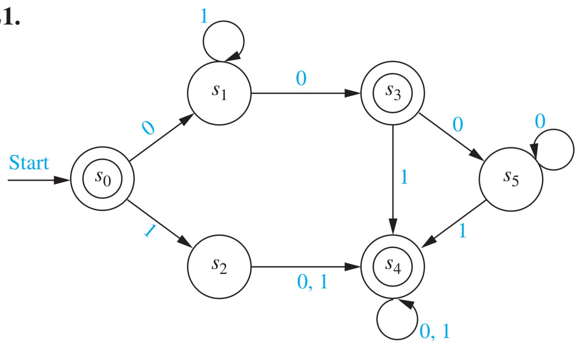
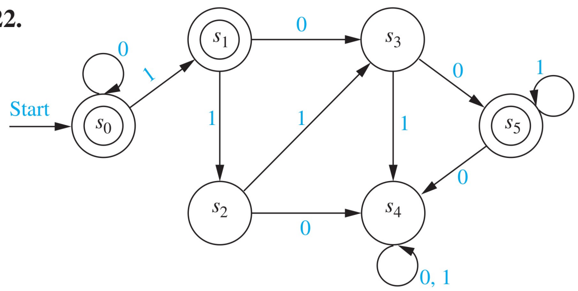


22.



23. Construct a deterministic finite-state automaton that recognizes the set of all bit strings beginning with 01.

24. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that end with 10.

25. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain the string 101.

26. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that do not contain three consecutive 0s.

27. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain exactly three 0s.

28. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain at least three 0s.

29. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain three consecutive 1s.

30. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that begin with 0 or with 11.

31. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that begin and end with 11.

32. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain an even number of 1s.

33. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain an odd number of 0s.

34. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain an even number of 0s and an odd number of 1s.

35. Construct a finite-state automaton that recognizes the set of bit strings consisting of a 0 followed by a string with an odd number of 1s.

36. Construct a finite-state automaton with four states that recognizes the set of bit strings containing an even number of 1s and an odd number of 0s.

37. Show that there is no finite-state automaton with two states that recognizes the set of all bit strings that have one or more 1 bits and end with a 0.

38. Show that there is no finite-state automaton with three states that recognizes the set of bit strings containing an even number of 1s and an even number of 0s.

39. Explain how you can change the deterministic finite-state automaton M so that the changed automaton recognizes the set $I^* - L(M)$.

40. Use Exercise 39 and finite-state automata constructed in Example 6 to find deterministic finite-state automata that recognize each of these sets.

a) the set of bit strings that do not begin with two 0s

b) the set of bit strings that do not end with two 0s

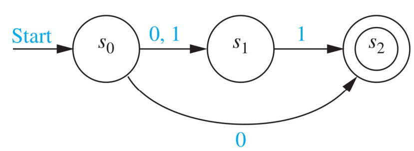
c) the set of bit strings that contain at most one 0 (that is, that do not contain at least two 0s)

41. Use the procedure you described in Exercise 39 and the finite-state automata you constructed in Exercise 25 to find a deterministic finite-state automaton that recognizes the set of all bit strings that do not contain the string 101.

42. Use the procedure you described in Exercise 39 and the finite-state automaton you constructed in Exercise 29 to find a deterministic finite-state automaton that recognizes the set of all bit strings that do not contain three consecutive 1s.

In Exercises 43–49 find the language recognized by the given nondeterministic finite-state automaton.

43.



44.