Kerry Ojakian's CSI 35 Class
Class Assignment #4 (and 5)

1. We recursively define f as follows: $f(1) = 5$ and $f(n+1) = 10 \cdot f(n) + 5$. Evaluate the following

   (a) $f(1)$

   (b) $f(3)$

   (c) $f(4)$

   (d) Describe $f(n)$ in words without recursion

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2. We recursively define g as follows: $g(1) = 1$ and $g(n+1) = (g(n))^2 + g(n)$. Evaluate the following

   (a) $g(2)$

   (b) $g(3)$

   (c) $g(4)$

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3. We recursively define f as follows: $f(0) = 9$ and $f(n+1) = -f(n)$. Evaluate the following

   (a) $f(3)$

   (b) $f(4)$

   (c) $f(1000)$

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4. Let $G(n) = 7^n$, defined on integers $n \geq 0$. Find a recursive definition for G.

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5. Give a recursive definition of the sequence a_n, $n \geq 1$, if $a_n = 8n$.

6. Let $F(n)$ be the number of strings of length n using just the letter b. Find a recursive definition for $F(n)$ (Warning: This question is so simple, it might be tricky! ...).

7. Let $F(n,k)$ be the number of strings of length n using letters in $\{a,b\}$ with exactly k b's. Find a recursive definition for $F(n,k)$.

8. Let $F(n,k)$ be the number of strings of length n using letters in $\{a,b,c\}$ with exactly k b's. Find a recursive definition for $F(n,k)$.

9. Give a recursive definition of the set of positive odd integers.

10. Give a recursive definition of the set of even integers (including zero and the negatives).

11. Consider the set S defined by $14 \in S$ and $s + t \in S$ whenever $s \in S$ and $t \in S$. Show that every element of S is even.

12. Let g be defined recursively by $g(0) = g(1) = 7$ and $g(n+1) = 2 \cdot g(n) + g(n-1)$.
Prove that $g(n)$ is always odd.

13. Define a subset of binary strings S as follows (implicit operation is concatenation): $101 \in S$ and if $x \in S$ then $x01 \in S$ and $x10 \in S$.

    (a) List all the length 7 binary strings in S

    (b) Prove that the strings in S all have one more 1 than 0.