SOLUTIONS

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

CSI 30 — Exam 3
Kerry Ojakian — SPRING 2026
YOUR NAME (first, then last):

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points (scaled to 100).

1. What are the quotient and remainder when

   (a) 23 is divided by 4.   (b) -8 is divided by 3.

   SOLUTION.

   (a): Quotient $= 5$. Remainder $= 3$. Because $23 = 4 \cdot 5 + 3$
   (b): Quotient $= -3$. Remainder $= 1$. Because $-8 = 3 \cdot (-3) + 1$

---

2. (a) What is the additive inverse of 5 (mod 9)?

   (b) What is the multiplicative inverse of 5 (mod 9)?

   SOLUTION.

   (a): 4. Since $5 + 4 \equiv 0 \pmod 9$
   (b): 2. Since $5 \cdot 2 \equiv 1 \pmod 9$

---

3. Consider the function $g : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$ given by the following table.

x	1	2	3	4	5
$g(x)$	3	1	5	2	4

Use the table to evaluate each (for evaluations, just give the answer).

(a) $g(2)$

(b) $g(5)$

(c) $g^{-1}(5)$

(d) $g^{-1}(1)$

a) Is g injective? b) Is g surjective? c) Is g bijective? d) Does g have an inverse?
e) If g has an inverse, find it?

SOLUTION.

a) $g(2) = 1$ b) $g(5) = 4$ c) $g^{-1}(5) = 3$ d) $g^{-1}(1) = 2$

a) Yes, g is injective (since different inputs are assigned different outputs).
b) Yes, g is surjective (since the range includes all of 1, 2, 3, 4, and 5).
c) Yes, g is bijective (since it is both injective and surjective).
d) Yes, g has an inverse (because it is bijective).
e) The inverse g^{-1} is given by:
$g^{-1}(1) = 2, \quad g^{-1}(2) = 4, \quad g^{-1}(3) = 1, \quad g^{-1}(4) = 5, \quad g^{-1}(5) = 3.$

---

4. Suppose $y \equiv 5 \pmod{11}$

(a) Calculate $7y \pmod{11}$

(b) Calculate $y^4 \pmod{11}$

SOLUTION.

$7y \equiv 7 \times 5 \equiv 35 \equiv 2 \pmod{11}$
$y^4 \equiv 5^4 \equiv 25 \cdot 25 \equiv 3 \cdot 3 \equiv 9 \pmod{11}$

---

5. Convert each number to decimal.

(a) The binary number 10101.

(b) The HEX number $1A9$.

SOLUTION.

(a): Decimal is $2^4 + 0 + 2^2 + 0 + 2^0 = 16 + 4 + 1 = 21$
(b): Decimal is $1 \times 16^2 + 10 \times 16^1 + 9 \times 16^0 = 256 + 160 + 9 = 425$

---

6. Find the sums of the following.

   (a) The binary numbers 11001 and 101.

   (b) The HEX numbers $1F4$ and $7C$.

   SOLUTION.

   In both cases we carry appropriately ...

   (a)
   11001
   + 101
   11110

   (b)
   $1F4$
   $+7C$
   270

---

7. Find the prime factorizations of 315 and 84, then use these factorizations to find the GCD and LCM. You may leave your final answer as a product of exponential expressions.

   SOLUTION.

   Factorizations are: $315 = 3^2 \cdot 5 \cdot 7$ and $84 = 2^2 \cdot 3 \cdot 7$
   GCD $= 2^0 \cdot 3^1 \cdot 5^0 \cdot 7^1 = 3 \cdot 7$
   LCM $= 2^2 \cdot 3^2 \cdot 5 \cdot 7$

---

8. For each pair of numbers, are they relatively prime? If not, find an integer that is relatively prime with both integers in the pair.

   (a) 14 and 25

   (b) 15 and 18

   SOLUTION.

   For 14 and 25, they are relatively prime since $\gcd(14, 25) = 1$.
   For 15 and 18, they are not relatively prime since $\gcd(15, 18) = 3$. An integer that is relatively prime with both 15 and 18 is 7 since $\gcd(15, 7) = 1$ and $\gcd(18, 7) = 1$.

---

9. Use a shift cipher with key 6 to encode the message "SUN". In your work, 1) Show the translations between letters and numbers (using the standard translation of 0 for 'A', 1 for 'B', and so on), 2) Show the encoding calculations with numbers, 3) Give your final encoded answer as a 3 letter word.

   SOLUTION.

   1) SUN translated to numbers is 18, 20, 13.
   2) $18 + 6 \equiv 24 \pmod{26}$
   $20 + 6 \equiv 0 \pmod{26}$
   $13 + 6 \equiv 19 \pmod{26}$
   3) Translating the numbers 24, 0, 19 back to letters we get: YAT

---

10. Let $C = \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$ and $D = \begin{bmatrix} 4 & 1 \\ 1 & 2 \\ 2 & 0 \end{bmatrix}$

For each, evaluate or state why it is undefined.

(a) $C \cdot D$

(b) $D \cdot C$

SOLUTION.

$C \cdot D$ is undefined because the number of columns in C (which is 2) does not match the number of rows in D (which is 3).

$$D \cdot C = \begin{bmatrix} 4 & 1 \\ 1 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} (4 \cdot 2 + 1 \cdot 0) & (4 \cdot 1 + 1 \cdot (-3)) \\ (1 \cdot 2 + 2 \cdot 0) & (1 \cdot 1 + 2 \cdot (-3)) \\ (2 \cdot 2 + 0 \cdot 0) & (2 \cdot 1 + 0 \cdot (-3)) \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 2 & -5 \\ 4 & 2 \end{bmatrix}$$

---

11. Suppose a, b, c, and d are integers, where $a \neq 0$. Prove that if $a|c$ and $b|d$, then $ab|cd$.

SOLUTION.

1) $c = a \cdot k$, for some integer k (by given: $a|c$)
2) $d = b \cdot m$, for some integer m (by given: $b|d$)
3) $cd = (ak)(bm) = (ab)(km)$ (by lines 1 and 2)
4) $ab|cd$ (by line 3, since $cd = (ab)(n)$ for integer $n = km$)
