SOLUTIONS

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

CSI 30 | Exam 2
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YOUR NAME (first, then last):

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points (total scaled to 100).

1. Suppose $A = \{2, 7, 8\}$, $B = \{2, 6\}$, and $C = \{6, 7, 8, 9\}$. Find the following.

   (a) $A \times B$ (b) $A \cap C$ (c) $(A \cup B) - C$

   SOLUTION.

   (a) $= \{(2, 2), (2, 6), (7, 2), (7, 6), (8, 2), (8, 6)\}$

   (b) $= \{7, 8\}$

   (c) $= \{2, 6, 7, 8\} - C = \{2\}$

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2. For each function $Z \times Z \to Z$, determine if it is i) onto?, ii) one-to-one?, iii) bijective?

   (a) $f(a, b) = a^6 + b^4 - 2$

   (b) $g(x, y) = x^3 + y$

   SOLUTION.
   1. Function $f(a, b) = a^6 + b^4 - 2$:
   i) *Onto:* The codomain is $\mathbb{Z}$. Consider $z \in \mathbb{Z}$. We need $a^6 + b^4 - 2 = z$, which implies $a^6 + b^4 = z + 2$.
   For $z \to -\infty$, $a^6 + b^4 \geq -2$ shows there exist integers a, b such that $a^6 + b^4 = n$ for any $n \geq -2$. However, no integer a, b satisfy $a^6 + b^4 = -3$, indicating not all z values are achievable. Therefore, f is not onto.
   ii) *One-to-one:* Consider $f(a_1, b_1) = f(a_2, b_2)$. We have $a_1^6 + b_1^4 = a_2^6 + b_2^4$, but this does not guarantee $a_1 = a_2$ and $b_1 = b_2$. For example, $f(0, 1) = f(1, 0) = -1$ but $(0, 1) \neq (1, 0)$. Therefore, f is not one-to-one.
   iii) *Bijective:* Since f is neither onto nor one-to-one, it is not bijective.
   2. Function $g(x, y) = x^3 + y$:
   i) *Onto:* The codomain is $\mathbb{Z}$. For any $z \in \mathbb{Z}$, let $x = 0$ and $y = z$. Then $g(0, y) = 0 + y = z$. Thus, for every z, there exists $(x, y) = (0, z)$ such that $g(x, y) = z$. Therefore, g is onto.
   ii) *One-to-one:* Consider $g(x_1, y_1) = g(x_2, y_2)$. We have $x_1^3 + y_1 = x_2^3 + y_2$. It requires $x_1^3 = x_2^3$ and $y_1 = y_2$, which implies $x_1 = x_2$. Thus, $(x_1, y_1) = (x_2, y_2)$, showing g is one-to-one.
   iii) *Bijective:* Since g is both onto and one-to-one, it is bijective.

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3. Write conditional statment that depends on the value of variable x. If x is positive, then x is set to 1, while if x is negative x is set to -1.
   Side question: What does your statement do if x is zero?
   SOLUTION.

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if x > 0:
  x = 1
elif x < 0:
  x = -1
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4. Let $X = \{u \in \mathcal{Z} \mid 1 \leq u \leq 100\}$ and $Y = \{u \in \mathcal{Z} \mid -100 \leq u \leq -1\}$. Let the function $h : X \to Y$ be defined by $h(n) = -101 + n$. Prove that h is bijective. Why does h have an inverse? What is $h^{-1}(-50)$?
SOLUTION.
To prove that the function $h : X \to Y$ defined by $h(n) = -101 + n$ is bijective, we need to show that it is both injective and surjective.
Injectivity:
Suppose $h(n_1) = h(n_2)$. Then:

$$-101 + n_1 = -101 + n_2$$

Adding 101 to both sides, we get:

$$n_1 = n_2$$

Therefore, h is injective.
Surjectivity:
Let $v \in Y$ such that $-100 \leq v \leq -1$. We need to find an $n \in X$ such that $h(n) = v$.
Set:

$$-101 + n = v$$

Solving for n, we get:

$$n = v + 101$$

Since $-100 \leq v \leq -1$, we have:

$$1 \leq v + 101 \leq 100$$

Thus, $n = v + 101 \in X$. Therefore, h is surjective.
Since h is both injective and surjective, h is bijective.
Inverse Function:
Because h is bijective, it has an inverse function $h^{-1} : Y \to X$.
Find $h^{-1}(-50)$:
Set $h(n) = -50$:

$$-101 + n = -50$$

Solving for n, we add 101 to both sides:

$$n = -50 + 101 = 51$$

Thus, $h^{-1}(-50) = 51$.

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5. Prove that $A - B = A \cap \overline{B}$, for sets A and B.
SOLUTION.
To prove that $A - B = A \cap \overline{B}$, consider the definitions of set difference and set complement: 1. **Definition of Set Difference**: $A - B = \{x \mid x \in A \text{ and } x \notin B\}$. 2. **Definition of Set Complement**: $\overline{B} = \{x \mid x \notin B\}$.
Proof:
Take any element x.
- If $x \in A - B$, then by definition: - $x \in A$ - $x \notin B$
Therefore, $x \in \overline{B}$.
Hence, $x \in A$ and $x \in \overline{B}$, which implies:

$$x \in A \cap \overline{B}$$

- Conversely, if $x \in A \cap \overline{B}$, then: - $x \in A$ - $x \in \overline{B}$ implies $x \notin B$
Therefore, $x \in A - B$.
Thus, both sets are equal:

$$A - B = A \cap \overline{B}$$

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6. Consider the function $f : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$ given by the following table.

x	1	2	3	4	5
$f(x)$	2	3	4	5	1

Use the table to evaluate each (for evaluations, just give the answer).

(a) $f(4)$

(b) $f(1)$

(c) $f^{-1}(4)$

(d) $f^{-1}(3)$

Is f injective? Is f surjective? Is f bijective?
SOLUTION.

$$f(4) = 5$$

$$f(1) = 2$$

$$f^{-1}(4) = 3$$

$$f^{-1}(3) = 2$$

Function f is injective because each element in the domain maps to a unique element in the codomain.
Function f is surjective because every element in the codomain has a pre-image in the domain.
Function f is bijective because it is both injective and surjective.

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7. Let $f(x) = 5 - 2x$ and $g(x) = 2 - x^2$.

(a) Evaluate $(f \circ g)(-5)$

(b) Simplify $(g \circ f)(x)$

SOLUTION.
$(f \circ g)(-5) = f(g(-5)) = f(2 - (-5)^2) = f(2 - 25) = f(-23) = 5 - 2(-23) = 5 + 46 = 51$
$(g \circ f)(x) = g(f(x)) = g(5-2x) = 2-(5-2x)^2 = 2-(25-20x+4x^2) = 2-25+20x-4x^2 = -23 + 20x - 4x^2$
