SOLUTION

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

CSI 30 Exam 1
Kerry Ojakian SPRING 2026
YOUR NAME (first, then last):

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points (scaled to 100).

1. Construct a truth table for the following proposition and determine whether it is a tautology, contradiction, or contingency: $(X \vee Y) \leftrightarrow (\neg X \wedge \neg Y)$
   SOLUTION.
   The proposition is a contradiction, since the final column is all False.

X	Y	$\neg X$	$\neg Y$	$X \vee Y$	$\neg X \wedge \neg Y$	$(X \vee Y) \leftrightarrow (\neg X \wedge \neg Y)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

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2. Write some code that first assigns 9 to the variable x, then increments x by 2.
   SOLUTION.
   One possible Python code is:

```
x = 9
x = x + 2
```

After executing the two statements, the value of x becomes 11.

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3. In Python, write the definition of a function F taking in an input (expected to be an integer). If the input is odd, then that same number is returned. If the input is even, then half of the number is returned.
Example: $F(20)$ should return 10, while $F(25)$ should return 25
SOLUTION.
A possible definition is:

```
def F(x):
   if x % 2 == 0:
      return x // 2
   else:
      return x
```

If x is even then $x/2$ is returned; otherwise the original value is returned.

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4. Let $Q(x, y)$ denote the statement: "$x^2 < y$ or $y = 0$". What are the truth values of the following?

(a) $Q(2, 7)$

(b) $Q(7, 2)$

(c) $Q(-3, 0)$

(d) $Q(0, 5)$

SOLUTION.

(a) $Q(2, 7)$: since $2^2 = 4$ and $4 < 7$, the statement is True.

(b) $Q(7, 2)$: since $7^2 = 49$ and $49 < 2$ is false and $y = 2 \neq 0$, the statement is False.

(c) $Q(-3, 0)$: although $(-3)^2 = 9$ is not less than 0, the second part $y = 0$ is true, so the statement is True.

(d) $Q(0, 5)$: since $0^2 = 0$ and $0 < 5$, the statement is True.

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5. Let $P(x, y)$ be the predicate: "$x^2 < y$". Suppose the domain is all integers. For each statement, is it true or false?

   (a) $P(3, 3)$ (b) $\exists x\ P(x, 0)$ (c) $\forall x\ \exists y\ P(x, y)$

   SOLUTION.

   (a) False. $P(3, 3)$ states that "$3^2 < 3$" which is false since $9 < 3$ is false.

   (b) False. Since for any integer x, $x^2 \geq 0$, so it cannot be that $x^2 < 0$.

   (c) True. For any integer x, one can take y to be $x^2 + 1$ or larger to ensure that $x^2 < y$ is true.

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6. Is this system specification consistent?

   - If students are being graded, professors cannot teach.
   - If professors can teach, then students can learn.
   - If students can learn, then students are not being graded.

   SOLUTION.
   Let

   $$G = \text{students are graded}, \quad T = \text{professors teach}, \quad L = \text{students learn}.$$

   The statements become

   $$G \to \neg T \quad T \to L \quad L \to \neg G$$

   The system is consisent. One such assignment that works is this:

   $$L = True \quad G = False \quad T = False$$

   Then 1) $G \to \neg T$ is True because T is False, 2) $T \to L$ is True because L is True, and 3) $L \to \neg G$ is True because G is False.

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7. Show that the following 2 propositions are logically equivalent: $\neg X$ and $\neg\neg\neg X$
SOLUTION.
Construct a truth table and compare the columns for the two propositions.

X	$\neg X$	$\neg\neg X$	$\neg\neg\neg X$
T	F	T	F
F	T	F	T

The column for $\neg X$ and the column for $\neg\neg\neg X$ are identical. Therefore the propositions are logically equivalent.

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8. Consider the following two inferences. For each, which inference rule from the "Inference Rules" sheet justifies it? In each case, you must state the name of the inference rule, and how the variables on sheet (some combination of p, q, r) match the inferences here.

(a) From $(\neg X) \to (\neg Y)$ and $\neg X$, conclude $\neg Y$.

(b) From $(\neg Y) \to (A \land B)$ and $X \to (\neg Y)$, conclude $X \to (A \land B)$.

SOLUTION.

(a) This is **Modus Ponens**. Match $p = \neg X$ and $q = \neg Y$. From $p \to q$ and p, we conclude q.

(b) This is **Hypothetical Syllogism**. Match $p = X$, $q = \neg Y$, and $r = A \land B$. From $p \to q$ and $q \to r$, we conclude $p \to r$.

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9. Suppose you are given the following premises:

$$\neg X, \quad P \wedge Q, \quad P \rightarrow (X \vee Y)$$

Conclude Y, by giving a structured proof where each line is justified by a named inference rule.
SOLUTION.

(a) $\neg X$ Premise

(b) $P \wedge Q$ Premise

(c) $P \rightarrow (X \vee Y)$ Premise

(d) P Simplification from (b)

(e) $X \vee Y$ Modus Ponens from (c) and (d)

(f) Y Disjunctive Syllogism using (e) and (a)

Thus Y follows from the premises.
