Kerry Ojakian's CSI 30 Class
Handout For Topic #6

The following are the *Set Theory Principles* you should use in a Set Theory proof.

1. $x \in A \cup B \iff x \in A \vee x \in B$ (definition of union)
2. $x \in A \cap B \iff x \in A \wedge x \in B$ (definition of intersection)
3. $x \in A - B \iff x \in A \wedge x \notin B$ (definition of difference)
4. $x \in \overline{A} \iff x \notin A$ (definition of complement)
5. $x \notin \emptyset$ (definition of empty set)

To do a proof, use the Set Theory Principles to convert between set theory statements and logic statements. Then use the inference rules (from Inference Rules sheet) and the logical equivalences (from the Logical Equivalences sheet) to do the proof. Recall that to prove $X = Y$ for sets you need 2 parts: 1) $X \subseteq Y$, and 2) $Y \subseteq X$.

Example:

- Prove that $A \cup \emptyset = A$.
  SOLUTION.
  Part 1 ($A \cup \emptyset \subseteq A$)
  1) $x \in A \cup \emptyset$ (premise)
  2) $x \in A \vee x \in \emptyset$ (definition of union from line 1)
  3) $x \notin \emptyset$ (definition of empty set)
  4) $x \in A$ (by Disjunctive Syllogism from lines 2 and 3)

  Part 2 ($A \subseteq A \cup \emptyset$)
  1) $x \in A$ (premise)
  2) $x \in A \vee x \in \emptyset$ (by Addition from line 1)
  3) $x \in A \cup x \in \emptyset$ (by definition of union from line 2)