

## HW #2

Kerry Ojakian's CSI 30 Class

Due Date: Tuesday March 24 (beginning of class)

**General Instructions:** Homework must be stapled, be relatively neat, and have your name on it. All work on this handout. Don't copy!

### The Assignment

1. Suppose we are in the universal set  $U = \{x \in Z \mid 1 \leq x \leq 15\}$ . Let  $E$  be the even integers from  $U$ . Let  $P = \{4, 5, 8, 9, 12, 13\}$ . Let  $Q = \{5, 6, 9, 10, 13, 15\}$ . Find the following.

(a)  $\overline{E} \cap P \cap \overline{Q}$

(b)  $(E \cup \overline{Q}) \cap \overline{P \cup Q}$

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2. (a) Suppose  $X = \{2, 3, 5\}$ . Find the power set of  $X$ .  
(b) Suppose  $Y = \{a \in Z \mid -10 < a < 10\}$ . How many elements are in the power set of  $Y$ ? (use an exponential expression to give your answer).

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3. Prove that  $A \cup B \subseteq B$
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4. Prove that  $A \cup (B - A) \subseteq A \cup B$

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5. Prove that  $A \cap (B - A) \subseteq \emptyset$

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6. Prove that  $A \cap \bar{A} = \emptyset$  for any set  $A$ .

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7. Prove that  $(A \cap B) \cup (A \cap \bar{B}) = A$  for sets  $A$  and  $B$ .

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8. Prove that  $A - B = A \cap \overline{B}$ , for sets  $A$  and  $B$ .

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9. Let  $g(x) = 4 + 3x^2$ . Evaluate  $g(2)$  and  $g(-2)$ .

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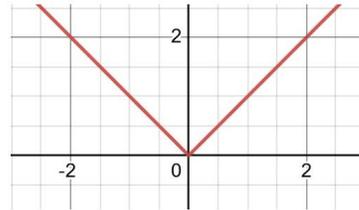
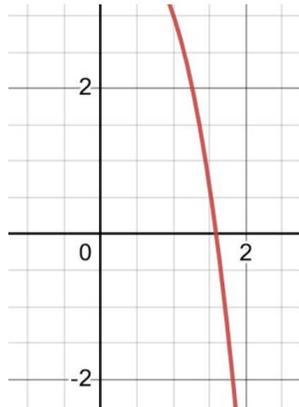
10. Consider the following two relations. For each one, is it a function or not?

(a)  $\{(2, 4), (4, 2), (3, 4), (1, 1)\}$

(b)  $\{(2, 4), (3, 5), (2, 6), (7, 9)\}$

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11. Consider functions graphed below. For each one, state whether or not it is one-to-one.





15. Let  $A = \{0, 2, 4, 6, 8\}$  and  $B = \{3, 4, 5, 6\}$ .

(a) Let the function  $h : A \rightarrow B$  be defined by:

$$h(0) = 6, \quad h(2) = 5, \quad h(4) = 4, \quad h(6) = 3, \quad h(8) = 6.$$

Is  $h$  surjective?

(b) Why is  $h$  not injective? Is there any injective function from  $A$  to  $B$ ?

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16. Let  $X = \{u \in \mathcal{Z} \mid 1 \leq u \leq 100\}$  and  $Y = \{u \in \mathcal{Z} \mid 10 \leq u \leq 109\}$ . Let the function  $h : X \rightarrow Y$  be defined by  $h(n) = n + 9$ . Prove that  $h$  is bijective. Why does  $h$  have an inverse? What is  $h^{-1}(109)$ ?

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17. Let  $X = \{u \in \mathbb{Z} \mid 1 \leq u \leq 10\}$ . Let the function  $h : X \rightarrow X$  be defined by  $h(n) =$

$$\begin{cases} n + 1 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

(a) Describe the range of  $h$ . Is  $h$  onto?

(b) Use the same definition for  $h$ , but now let  $X = \{u \in \mathbb{Z} \mid 1 \leq u \leq 100\}$ . Describe the new range.

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18. For each function  $Z \times Z \rightarrow Z$ , determine if it is i) onto?, ii) one-to-one?, iii) bijective?

(a)  $f(a, b) = a - b$

(b)  $g(x, y) = x^2 + x^4$