SOLUTION

BRONX COMMUNITY COLLEGE

of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

CSI 30 | Exam 3
Kerry Ojakian | FALL 2025
YOUR NAME (first, then last):

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points (scaled to 100).

1. What are the quotient and remainder when

   (a) 17 is divided by 3. (b) -5 is divided by 2.

   SOLUTION:
   (a): Quotient $= 5$. Remainder $= 2$. Because $17 = 3 \cdot 5 + 2$
   (b): Quotient $= -3$. Remainder $= 1$. Because $-5 = 2 \cdot (-3) + 1$

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2. (a) What is the additive inverse of 4 (mod 7)?

   (b) What is the multiplicative inverse of 4 (mod 7)?
   SOLUTION:
   (a): 3. Since $4 + 3 \equiv 0 \pmod 7$
   (b): 2. Since $4 \cdot 2 \equiv 1 \pmod 7$

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3. Suppose $x \equiv 3 \pmod{10}$

(a) Calculate $4x \pmod{10}$

(b) Calculate $x^{16} \pmod{10}$

SOLUTION.
(a): $4 \cdot 3 = 12 \equiv 2 \pmod{10}$. Answer: 2.
(b): Answer is 1, because mod 10 we calculate (could do faster with modular exp):
$3^2 = 3 \cdot 3 = 9, \quad 3^3 = 9 \cdot 3 \equiv 27 \equiv 7, \quad 3^4 \equiv 7 \cdot 3 \equiv 21 \equiv 1, \quad 3^5 \equiv 1 \cdot 3 \equiv 3$
$3^6 \equiv 3 \cdot 3 \equiv 9, \quad 3^7 \equiv 9 \cdot 3 \equiv 27 \equiv 7, \quad 3^8 \equiv 7 \cdot 3 \equiv 21 \equiv 1, \quad 3^9 \equiv 1 \cdot 3 \equiv 3$
$3^{10} \equiv 3 \cdot 3 \equiv 9, \quad 3^{11} \equiv 9 \cdot 3 \equiv 27 \equiv 7, \quad 3^{12} \equiv 7 \cdot 3 \equiv 21 \equiv 1, \quad 3^{13} \equiv 1 \cdot 3 \equiv 3$
$3^{14} \equiv 3 \cdot 3 \equiv 9, \quad 3^{15} \equiv 9 \cdot 3 \equiv 27 \equiv 7, \quad 3^{16} \equiv 7 \cdot 3 \equiv 21 \equiv 1$

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4. Convert each number to decimal.

(a) The binary number 11010.

(b) The HEX number $10E$.

SOLUTION:
(a): Decimal is $2^4 + 2^3 + 0 + 2^1 + 0 = 16 + 8 + 2 = 26$
(b): Decimal is $16^2 + 0 + 14 = 256 + 14 = 270$

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5. Convert the decimal number 45 to binary.
SOLUTION:
Answer: 101101 by the following calculation (put first digit in ones spot etc).
$45 = 22 \cdot 2 + 1 \quad 22 = 11 \cdot 2 + 0 \quad 11 = 5 \cdot 2 + 1 \quad 5 = 2 \cdot 2 + 1 \quad 2 = 1 \cdot 2 + 0 \quad 1 = 0 \cdot 2 + 1$

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6. Find the sums of the following.

(a) The binary numbers 10110 and 11.

(b) The HEX numbers $2A9$ and $5E$.

SOLUTION:
In both cases we carry appropriately ...

(a)	(b)
10110	$2A9$
+ 11	$+5E$
11001	307

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7. Find the prime factorizations of 450 and 54, then use these factorizations to find the GCD and LCM.
SOLUTION:
Factorizations are: $450 = 2 \cdot 3^2 \cdot 5^2$ and $54 = 2 \cdot 3^3$
GCD $= 2^1 \cdot 3^2 \cdot 5^0 = 18$
LCM $= 2^1 \cdot 3^3 \cdot 5^2$

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8. For each pair of numbers, are they relatively prime? If not, find an integer that is relatively prime with both integers in the pair.

   (a) 20 and 27 (b) 10 and 9

   SOLUTION:
   (a): 20 and 27 are relatively prime since their GCD is 1.
   (b): 10 and 9 are relatively prime since their GCD is 1.

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9. Use a shift cipher with key 4 to encode the message "WIN". In your work, 1) Show the translations between letters and numbers (using the standard translation of 0 for 'A', 1 for 'B', and so on), 2) Show the encoding calculations with numbers, 3) Give your final encoded answer as a 3 letter word.
   SOLUTION:
   1) WIN translated to numbers is 22, 8, 13.
   2) $22 + 4 \equiv 0 \pmod{26}$ $\quad 8 + 4 \equiv 12 \pmod{26}$ $\quad 13 + 4 \equiv 17 \pmod{26}$
   3) Translating the numbers 0, 12, 17 back to letters we get: AMR

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10. Let $X = \{u \in \mathcal{Z} \mid 1 \leq u \leq 100\}$ and $Y = \{u \in \mathcal{Z} \mid 10 \leq u \leq 109\}$. Let the function $h : X \to Y$ be defined by $h(n) = n + 9$. Prove that h is bijective. Why does h have an inverse? What is $h^{-1}(109)$?
    SOLUTION:
    $h^{-1}(109) = 100$ since $h(100) = 109$.

    h is injective because: $h(x) = h(y) \to x + 9 = y + 9 \to x = y$
    h is surjective because: For any y where $10 \leq y \leq 109$, we can find an x so that $h(x) = y$, by letting $x = y - 9$. Note that $1 \leq x \leq 100$, so x is in the domain of h. Furthermore, $h(x) = y - 9 + 9 = y$.

    Since h is injective and surjective, by definition, h is bijective. Since h is bijective, a theorem tells us that h has an inverse.
