SOLUTIONS

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

CSI 30
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YOUR NAME (first, then last):

Exam 2
FALL 2025

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points.

1. Write a for-loop loop that sums the numbers from 1 to 100.

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A=0
for x in range(1,101):
    A=A+x
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2. Suppose n is an integer. Prove n is odd if and only if $n+101$ is even.

($\Rightarrow$) Assume n odd
1. Thus $n=2k+1$, some integer k
2. Thus $n+101=2k+1+101=2(k+51)$
3. Thus $n+101$ is even

($\Leftarrow$) Assume $n+101$ is even
1. Thus $n+101=2k$, some integer k
2. Thus $n=2k-101=2(k-51)+1$
3. Thus n is odd.

3. Consider the function $f : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$ given by the following table.

x	1	2	3	4	5
$f(x)$	2	4	3	5	1

Use the table to evaluate each (for evaluations, just give the answer).

(a) $f(4) = 5$

(b) $f(1) = 2$

(c) $f^{-1}(4) = 2$

(d) $f^{-1}(3) = 3$

Is f injective? Is f surjective? Is f bijective? → YES, since both injective + surject

YES

Since different inputs are assigned different outputs

YES

All of coDomain (1,2,3,4,5) is among outputs.

4. Let $f(x) = -9 - 3x$ and $g(x) = 2 + x^3$.

(a) Evaluate $(g \circ f)(-2)$

$= g(f(-2))$

$= g(-9-3(-2))$

$= g(-3) = 2 + (-3)^3$

$= 2 - 27 = \boxed{-25}$

(b) Simplify $(f \circ g)(x)$

$= f(g(x)) = f(2+x^3)$

$= -9 - 3(2+x^3)$

$= -9 - 6 - 3x^3$

$= \boxed{-15 - 3x^3}$

5. Let $A = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$

For each, evaluate or state why it is undefined.

(a) $A \cdot B$

Undefined, since

columns $A = 2$

$\neq 3 = $ # rows B

(b) $B \cdot A = \begin{bmatrix} 3\cdot1+0\cdot(-2) & 3\cdot2+0\cdot0 \\ 0\cdot1+2\cdot(-2) & 0\cdot2+2\cdot0 \\ 1\cdot1+3\cdot(-2) & 1\cdot2+3\cdot0 \end{bmatrix}$

$= \begin{bmatrix} 3 & 6 \\ -4 & 0 \\ -5 & 2 \end{bmatrix}$

6. Let $M = \begin{bmatrix} 1 & 1 & 0 \\ -3 & 0 & 3 \end{bmatrix}$ and $N = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & -2 \end{bmatrix}$

(a) Find $M + N$ $\longrightarrow$ $\begin{bmatrix} 1+2 & 1+0 & 0+0 \\ -3+0 & 0+(-1) & 3+(-2) \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ -3 & -1 & 1 \end{bmatrix}$

(b) What is the value of $M_{2,1} + N_{1,1}$?

(c) Use the subscript notation to name the entries of M and N which are zero.

b) $= -3 + 2$

$= \boxed{-1}$

c) $M_{1,3}$ $M_{2,2}$

$N_{1,2}$ $N_{1,3}$ $N_{2,1}$

$h^{-1}(109) = 100$ since $h(100) = 109$

7. Let $X = \{u \in \mathcal{Z} \mid 1 \le u \le 100\}$ and $Y = \{u \in \mathcal{Z} \mid 10 \le u \le 109\}$. Let the function $h : X \to Y$ be defined by $h(n) = n + 9$. Prove that h is bijective. Why does h have an inverse? What is $h^{-1}(109)$?

Injective: $h(x) = h(y)$

$$\Downarrow$$

$$x + 9 = y + 9$$

$$\Downarrow$$

$$x = y$$

Surjective

Given $10 \le y \le 109$ need x so that $h(x) = y$, that is $x + 9 = y$

so take $x = y - 9$ and note $1 \le x \le 100$

Bijective: Since injective and surjective

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8. Let $X = \{u \in \mathcal{Z} \mid 1 \le u \le 100\}$. Let the function $f : X \to X$ be defined by

$$f(n) = \begin{cases} n \text{ if } n \text{ is odd} \\ n - 1 \text{ if } n \text{ is even} \end{cases}$$

(a) Is f injective?

(b) Describe the range of f. Is f onto?

(c) Does f have an inverse function?

a) No [take any odd and the even that is +1 larger for example $f(5) = 5 = f(6)$]

b) Odds between 1 and 99 (since each odd maps to itself and each even maps to an odd)

Not onto [Misses all evens]

c) No. Since not onto $\Rightarrow$ not bijective $\Rightarrow$ no inverse