

CSI 30 LECTURE NOTES (Ojakian)

Topic 15: Counting

OUTLINE

(References: 6.1)

1. Product rule and tree diagrams
 2. Sum rule and Inclusion/exclusion.
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1. Basic Product Rule and Tree diagrams

- (a) Examples: If two different colored 3-sided dice are rolled, how many outcomes are possible? What about three dice? [make tree diagrams]
- (b) Comment on Tree Diagram:
 - i. Start with single root vertex.
 - ii. Make branch leaving each vertex for each allowed option *at that point*.
- (c) Theorem: $|A_1 \times \cdots \times A_k| = |A_1| \cdot |A_2| \cdots |A_k|$
- (d) More Examples.
 - i. How many binary strings of length 4? Of length n ?
 - ii. How many subsets of $[4]$? Of $[n]$?
 - iii. How many passwords of length 8? (if each character can be a digit, or upper/lower case letter).
If a hacker spends a day guessing 10 passwords a second, what fraction of the passwords have they checked?
 - iv. How many different standard NY State license plates are possible? (3 capital letters, followed by 4 digits)

2. Product Rule - Harder Problems

- (a) Example: Suppose 5 cards are in a bowl, numbered 1 thru 5.
 - i. If you select two cards (with replacement), how many outcomes are possible? So what is the probability you get a 3 and a 4 (in either order)? What is the probability that the cards have the same value?
 - ii. If you select two cards (without replacement), how many outcomes are possible? [draw tree diagram]
Consider last two probability questions now?
- (b) Example 22. How many bit strings of length 4 do not have 2 consecutive 1s

3. Sum Rule

- (a) Example: Suppose there are 30 professors and 50 students in a room. We will choose one person from the room. How many choices do we have?
- (b) Sum Rule: If the sets do NOT overlap, then:

$$|A_1 \cup \cdots \cup A_k| = |A_1| + \cdots + |A_k|$$

4. Subtraction and Inclusion-Exclusion

- (a) Example: Suppose a park has a total of 100 living creatures. Suppose 15 are rats, 20 are roaches, 10 are dogs, and there is one pigeon. The rest are humans. How many humans are in the park?
- (b) Example: A gym has 5 students on the basketball team, and 7 students on the baseball team. Can you determine how many students are in the gym? What if 3 are on both the basketball and baseball team?
- (c) $|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$
- (d) Example 19. With sets of applicants.
- (e) Example 18. Certain bit strings.

5. Exercises

- (a) How many functions are there from set with m elements to a set with n elements? What if we require the functions to be injective?
- (b) Section 6.1: 1 - 17, 21 - 37, 40 - 51
- (c) DNA Sequences. Section 6.1: 18, 19 (note: 4 choices for each: A, C, G, T).
- (d) Probability. Section 7.1: 7, 21, 35 - 37