

CSI 30 LECTURE NOTES (Ojakian)

Topic 6: Functions

OUTLINE

(References: 2.3)

1. Functions as kinds of relations

1. Definition of Function

(a) Typical imprecise definition:

A function f from set A to set B is an “assignment” of exactly one element of B to each element of A .

(b) Why?

- i. The pairs (t, y) such that t ranges from the year 2000 up to the present year, and for any t , we have that y is the number of enrolled CUNY students in that year.

(c) Precise definition using set products.

- i. Relation on $A \times B$.

- ii. Function definition:

A relation on $A \times B$ that contains one and only one ordered pair (a, b) for every element $a \in A$, defines a function f from A to B .

(d) Example represented in following ways

- i. List pairs (Class makes it up ...)
- ii. Table
- iii. Graph on the plane (and Vertical Line Test)
- iv. Two sides with lines/arrows

(e) More Examples (which are functions?)

- i. All pairs of integers (x, y) such that $y = x^3 + 1$
- ii. $<$ on the rationals
- iii. (x, x^2) for x real
- iv. (x^2, x) , for x real.

2. Function Terminology

(a) Function = Map = Mapping = Transformation ...

(b) Image = output = value

(c) Domain

(d) Codomain and Range

- i. Why codomain? Example: function $f : Z \rightarrow Z$ defined by $f(n) =$ [choose some polynomial]. So codomain clear, but range??

3. Function Evaluation

- (a) From algebraic definition
- (b) From table definition
- (c) From graph definition

4. Function Types

- (a) Injective (or one-to-one)
 - i. Horizontal Line Test
 - ii. Showing that a function is *not* injective: Find counter-example (example: $f(x) = x^2$)
- (b) Surjective (or onto)
 - i. Showing that a function is *not* surjective: Find counter-example (example: $f : Z \rightarrow Z$ defined by $f(n) = 3n$)
- (c) Bijective (or one-to-one correspondance)
 - i. Example: $f : Even \rightarrow N$, $f(n) = n/2$ and cardinality significance.

5. Building New Functions

- (a) Function Composition (picture in Example 23).
 - i. Do f then g : 2 notations - $g(f(x))$ or $g \circ f$
- (b) Inverse Function (exists iff bijective)
 - i. f and g inverses iff $f \circ g = g \circ f$

6. Exercises

Section 2.3: 2 - 23, 29, 35