CSI 30 LECTURE NOTES (Ojakian)

Topic 1: Introduction and Propositional Logic

OUTLINE
(References: 1.1, 1.2, 1.3)

1. Introduction
2. Propositional Logic
3. Applications and Terminology

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1. Math and Computer Science
   (a) Discrete math versus other math
   (b) Computer programming - mathematics in action
   (c) How do we make imprecise world precise for a machine? Logic! (our starting point)
2. General Mathematical Introduction
   (a) Basic development in mathematics:
      i. Definition.
      ii. Theorem (or Lemma or Claim or ...)
      iii. Proof
   (b) Example:
      i. How might you define the terminology "even" and "odd"?
      ii. Theorem: The addition of two evens is even.
   (c) Example:
      i. DEF: a *Word* is any finite sequence of the 26 letters from the English alphabet.
      ii. DEF: Say a word is *BRONX* if its length is odd, its first letter is B and its last letter is X.
      iii. DEF: Given any words, their *concatenation* is the formation of a single word by putting the words one after another.
      iv. Theorems? - Concatenating 2 BRONX words is BRONX? Etc?
3. Propositions
   (a) *Proposition*: sentence that declares a fact (is either true or false)
   (b) Examples: See Rosen (8th Ed): Section 1.1.2 (page 2)
4. Propositional Logic: Propositional Letters and Connectives
   (a) Makes thinking about propositions formal and precise
   (b) Use letters to represent propositions - each can be true (T) or false (F)
   (c) Have *connectives*: OR, AND, NOT, IMPLICATION, EQUIVALENCE

(d) Examples of connectives in English: See section 1.1.2 (pages 3 - 5, 7)

(e) Consider some unusual examples:

i. Absurd implications! (page 8)

ii. Unusual OR: "I open the door OR you open the door"

(f) Truth Table Definitions of connectives
(**and do in Python, where implication is a function**)

(g) Question: When does order of the propositional variables matter?

(h) DEF: The *converse* of an implication is ...

5. Propositional Logic: Compound Propositions

(a) Truth Tables

(b) **Do it in Python**

6. Propositional Logic: Terminology

(a) Tautology

i. Example: $A \vee \neg A$

ii. Example: $A \rightarrow (A \vee B)$

(b) Contradiction

i. Example: $X \wedge \neg X$

(c) Consistent

i. Example: $(P \vee Q) \rightarrow (P \wedge Q)$

7. Propositional Equivalences

(a) Example: $\neg\neg P$ equivalent to P

(b) Example: $\neg(P \vee Q)$ equivalent to $\neg P \wedge \neg Q$

(c) Rules

i. Double Negation: $\neg\neg P \equiv P$

ii. De Morgan's Law

iii. Implication equivalence: $A \rightarrow B \equiv \neg A \vee B$

iv. Equivalence (or if and only if): $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$

8. Propositional Logic Applications

(a) System Specification

i. Convert to propositional logic. Then find an assignment which makes the conjunction true.

ii. Do Example 4 and 5 in Section 1.2 (page 18)

(b) Circuits

i. Do Example 10 in Section 1.2 (page 22)

ii. Do Example 11 in Section 1.2 (page 23)