HW #2

Kerry Ojakian's CSI 30 Class
Due Date: Tuesday October 28 (beginning of class)

General Instructions: Homework must be stapled, be relatively neat, and have your name on it. All work on this handout. Don't copy!

The Assignment

1. Suppose we are in the universal set $U = \{x \in Z \mid 1 \leq x \leq 15\}$. Let E be the even integers from U. Let $P = \{4, 5, 8, 9, 12, 13\}$. Let $Q = \{5, 6, 9, 10, 13, 15\}$. Find the following.

   (a) $\overline{E} \cap P \cap \overline{Q}$

   (b) $(E \cup \overline{Q}) \cap \overline{P \cup Q}$

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2. (a) Suppose $X = \{2, 3, 5\}$. Find the power set of X.

   (b) Suppose $Y = \{a \in Z \mid -10 < a < 10\}$. How many elements are in the power set of Y? (use an exponential expression to give your answer).

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3. Prove that $A \cap \overline{A} = \emptyset$ for any set A (do it from the definitions).

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4. Prove that $(A \cap B) \cup (A \cap \overline{B}) = A$ for sets A and B (do it using a membership table).

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5. Prove that $A - B = A \cap \overline{B}$, for sets A and B (do it using Venn Diagrams).

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6. Let $g(x) = 4 + 3x^2$. Evaluate $g(2)$ and $g(-2)$.

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7. Consider the function f given by the following table.

x	2	4	6	10	15	17
$f(x)$	3	4	13	2	6	14

Use the table to evaluate each.

(a) $f(2)$

(b) $f(10)$

(a) $f^{-1}(14)$

(b) $f^{-1}(4)$

(a) $f^{-1}(2)$

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8. Let $f(x) = 2x$ and $g(x) = x + 10$.

   (a) Find $(f \circ g)(5)$

   (b) Find $g(f(5))$

   (c) Find $f(f(5))$

   (d) Find $(g \circ g)(5)$

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9. Given $f(x) = 2x + 1$ and $g(x) = x^2 + 3$, find and simplify the following.

   (a) $(f \circ g)(3)$

   (b) $(f \circ g)(x)$

   (c) $(g \circ f)(x)$

   (d) $(f \circ g \circ f)(x)$

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10. For each pair of functions, check if they are inverses by verifying $f(g(x)) = x$ and $g(f(x)) = x$.

    (a) $f(x) = 2x,\ g(x) = x + 2$

    (b) $f(x) = 7 - x,\ g(x) = 7 - x$

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11. Show that $f(x) = -3x + 1$ and $g(x) = \dfrac{x-1}{-3}$ are inverses.

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12. Let $A = \{0, 2, 4, 6, 8\}$ and $B = \{0, 1, 2, 3, 4, 5, 6\}$.

   (a) Let the function $f : A \to B$ be defined by $f(n) = n/2$. Is f injective?

   (b) Why is f not surjective? Is there any surjective function from A to B?

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13. Let $A = \{0, 2, 4, 6, 8\}$ and $B = \{3, 4, 5, 6\}$.

   (a) Let the function $h : A \to B$ be defined by:

   $$h(0) = 6, \quad h(2) = 5, \quad h(4) = 4, \quad h(6) = 3, \quad h(8) = 6.$$

   Is h surjective?

   (b) Why is h not injective? Is there any injective function from A to B?

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14. Let $X = \{u \in Z \mid 1 \leq u \leq 100\}$ and $Y = \{u \in Z \mid 10 \leq u \leq 109\}$. Let the function $h : X \to Y$ be defined by $h(n) = n + 9$. Prove that h is bijective. Why does h have an inverse? What is $h^{-1}(109)$?

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15. Let $X = \{u \in Z \mid 1 \leq u \leq 10\}$. Let the function $f : X \to X$ be defined by $h(n) = \begin{cases} n+1 \text{ if } n \text{ is odd} \\ n/2 \text{ if } n \text{ is even} \end{cases}$

    (a) Describe the range of h. Is h onto?

    (b) Use the same definition for h, but now let $X = \{u \in Z \mid 1 \leq u \leq 100\}$. Describe the new range.

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16. For each function $Z \times Z \to Z$, determine if it is i) onto?, ii) one-to-ont?, iii) bijective?

    (a) $f(a, b) = a - b$

    (b) $g(x, y) = x^2 + x^4$

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17. Find two 2 by 2 matrices M and N such that $M \cdot N = N \cdot M$.

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18. Consider the matrix $M = \begin{bmatrix} 11 & 2 & 8 & 9 \\ 6 & 4 & 13 & -1 \\ -7 & 5 & -3 & 0 \end{bmatrix}$

(a) What are the dimensions of M?

(b) What is the value at entry $M_{2,3}$

(c) Using the "$M_{i,j}$" notation, name the entries which are negative.

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19. Let $M = \begin{bmatrix} 1 & 12 & 0 & 9 \\ -6 & 0 & 3 & -1 \end{bmatrix}$ and $N = \begin{bmatrix} 11 & 2 & 8 & -9 \\ 0 & 4 & -2 & -1 \end{bmatrix}$

(a) Find $M + N$

(b) Find $M - N$

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20. Let $M = \begin{bmatrix} 1 & 2 \\ -6 & 0 \end{bmatrix}$ and $N = \begin{bmatrix} 3 & 2 \\ 0 & 4 \end{bmatrix}$

(a) Find $M \cdot N$

(b) Find $N \cdot M$
