HW #1

Kerry Ojakian's CSI 30 Class
Due Date: Thursday Sept 18 (beginning of class)

General Instructions:

- Homework must be stapled, be relatively neat, and have your name on it.
- Use tutors, work with other students, but ... don't copy!

The Assignment

1. Read the first 3 sections of the Krantz proof article (at webpage next to Topic 3). Write a brief summary of it (a page). Write about one point you agree with and one point you disagree with, and why. Do **not** use any AI assistance. Attach your typed response.

2. Show that the following 2 propositions are logically equivalent:
   $A \leftrightarrow B$ and $(A \rightarrow B) \wedge (B \rightarrow A)$

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3. Use De Morgan's laws to find the negation of each of the following statements.

   (a) $\neg A \wedge B$

   (b) BCC is in CUNY or BCC is not in NYC

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4. Are these system specifications consistent?

   - If students are being graded, professors cannot teach.
   - If professors can teach, then students can learn.
   - If students can learn, then students are not being graded.

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5. Determine the truth value of each of these statements if the domain consists of all integers

   (a) $\forall x \ (x < 2x)$

   (b) $\exists u \ (3u + 7 = 34)$

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6. Let $N(x)$ be the predicate: "x lives in NY".
   Let $B(y)$ be the predicate: "y lives in The Bronx".
   Express each of the following statements in terms of N and B, quantifiers, and logical connectives. Let the domain consist of residents of humans.

   (a) Someone lives in NY.

   (b) Everyone who lives in The Bronx lives in NY.

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7. Prove that the sum of any three odd integers is odd.

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8. Prove or disprove: If $a|bc$ then $a|b$ or $a|c$.

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9. Suppose n is an integer. Prove n is odd if and only if $n + 101$ is even.

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10. We know that the product of 2 consecutive positive integers is divisible by 2, and that the product of any 3 consecutive positive integers is divisible by 3.
Make a conjecture about k integers. Prove it!

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11. Prove that a right triangle *cannot* have all its side lengths equal to prime numbers (Hint: use Pythagorean Theorem and proof by contradiction).

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12. A **perfect square** is an integer that is equal to the square of another integer; for example 9 is a perfect square because $9 = 3^2$, but 18 is *not* a perfect square.

    (a) Prove that if m and n are both perfect squares, then nm is also a perfect square.

    (b) Prove or disprove: The sum of two perfect squares is a perfect square.

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13. List the members of these sets.

    (a) $\{x \mid x \text{ is a negative integer larger than } 5\}$

    (b) $\{y \mid y \text{ is a an integer such that } y^2 = -y\}$

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15. For each pair of sets, determine if one is a subset of the other (say which) or if neither is a subset of the other.

   (a) $A =$ set of positve integers divisble by 10. $B =$ set of positive even integers.

   (b) $X =$ students below the age of 20. $Y =$ students in college.

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16. Find the cardinality of each set. If it is infinite, just say infinite.

   (a) $\{1, 4, 2, 2\}$

   (b) $\{x \mid x \text{ is an integer and } (x-2)(x+2)(x-\frac{1}{2}) = 0\}$

   (c) The even integers larger than 100.

   (d) $\{a, \{2, 3\}, \{\{4.8\}, x\}, z\}$

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17. Suppose $A = \{1, 2, 3, 5, 6, 9\}$ and $B = \{6, 7, 8, 9\}$. Find the following.

   (a) $A \cup B$

   (b) $A \cap B$

   (c) $A - B$

   (d) $B - A$

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18. Suppose $A = \{x, y, z\}$ and $B = \{2, 6\}$. Find the following.

    (a) $A \times B$

    (b) $B \times A$

    (c) $B \times B$

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19. Suppose $X = \{1, 2, 5, 6, 7\}$, $Y = \{2, 3, 6, 7\}$, and $Z = \{2, 6, 9\}$. Find the following.

    (a) $(X \cap Y) \cup Z$

    (b) $X \cap (Y \cup Z)$

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20. Suppose we are in the universe $U = \{x \mid x \text{ is an integer such that } 1 \leq x \leq 9\}$. Suppose $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 7\}$. Find the following and draw Venn Diagrams for each.

    (a) $\overline{A}$

    (b) $\overline{B}$
