Kerry Ojakian's CSI 30 Class
Class Assignment #3

1. Suppose you are given the following premises:

   (a) "It is sunny"

   (b) "If it is sunny then it is daytime"

   (c) "If it is daytime then you should study"

   Conclude from these premises that "You should study". Do this by translating the phrases into formal logic. Then give a proof.

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2. Suppose you are given the following premises:

   (a) "NYC is not the best city in the world"

   (b) "If there were no homeless in NYC, then NYC would be the best city in the world"

   (c) "If there are homeless in NYC, then taxes in NYC should increase"

   Conclude from these premises that "Taxes in NYC should increase". Do this by translating the phrases into formal logic. Then give a proof.

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3. Suppose you are given the following premises:

   (a) "BCC is a university"

   (b) "Every university is on earth"

   Conclude from these premises that "BCC is on earth". Do this by translating the phrases into formal logic (using quantifiers), using the following predicates (whose domain are all buildings): $U(x)$ says "x is a university" and $E(x)$ says "x is on earth".

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4. Suppose you are given the following premises:

   (a) Every person is either politically engaged or rich (or both)

   (b) Bob is not rich.

   Conclude from these premises that Bob is politically engaged. Do this by translating the phrases into formal logic (using quantifiers), letting $P(x)$ mean "x is politically engaged" and $R(x)$ mean "x is rich".

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5. Consider summing two odd integers. What is your conjecture about the result? (even or odd or something else?) Prove it.

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6. Prove that the product of any two consecutive integers is even.

7. Prove or disprove: If n is an odd integer then $\frac{n-1}{2}$ is odd.

8. Prove that for all positive integers n, n^3 is odd if and only if n is odd.

9. Prove that the product of any 3 consecutive positive integers is divisible by 3.

10. Prove that for all positive integers n, n^2 is odd if and only if n is odd.

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11. Prove that if $A|B$ and $B|C$, then $A|C$.

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12. One of the following is true, and the other false (notice they are converses of eachother). Find a counter-example to the false claim, and prove the true one.

    (a) If $ab|c$, then both $a|c$ and $b|c$.

    (b) If both $a|c$ and $b|c$, then $ab|c$.

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13. Prove or disprove: If $a|b$ then $a^2|b^2$.

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14. Prove that 20 is *not* a perfect square (use cases).

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15. Prove that if x and y are integers and xy is even, then x is even or y is even (Hint: Use Proof-by-Contradiction)

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16. Consider the following program:

    if $x > 5$:
        $z = x + 1$
    else:
        $z = x - 1$

    Suppose the initial assertion is that $x = 3$. Determine what the final value of z will be, and prove it.

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17. Consider the following program:

    for x in range(1, 1000):
        $y = y + 3$

    Given that the initial value of y is even, prove that the final value is odd.

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18. Consider the following program:

    $a = 1$
    while $a > 0$:
        $a = a + 1$

    Prove that the program does *not* terminate.
