THIRD EXAM, MATH 31: CALCULUS. FALL 2013

Do all questions and show all the work. The total number of points in the test is 111.
Due 11/25/2013

1. (5 points) Find the limit $\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$.
2. (5 points) Find the value of c such that the function:

$$f(x) = \begin{cases} cx^2 + 2x, & x < 2 \\ x^3 - cx, & x \geq 2 \end{cases}$$

   is continuous on the real numbers $(-\infty, \infty)$.

3. (10 points) Given the polynomial function $f(x) = 3x^2 - 4x$:
   (a) Find the slope of the tangent line at the point $x_0 = 1$ **using the definition**.
   (b) Check your answer evaluating the derivative $f'(x)$ of $f(x)$ at $x_0 = 1$.
   (c) Find the equation of the tangent line at $(x_0, f(x_0))$.
4. (20 points) Find the derivative of the following functions:
   (a) $f(x) = 4\sin(3x^2 - 2x)$ $\quad(a')$ $f(x) = (x^3 + 2x)(2x^4 - 1)$
   (b) $f(x) = 2\cos(-4x + 7x^2)$ $\quad(b')$ $f(x) = \frac{x^3+2x}{2x^4-1}$
   (c) $f(x) = x(\ln(x) + x)$ $\quad(c')$ $f(x) = e^{\frac{1}{5}x^5+3x}$
   (d) $f(x) = \ln(3 + 2x^2)$ $\quad(d')$ $f(x) = \sqrt{-9x-2}$
   (e) $f(x) = \tan(3x - 1)$ $\quad(e')$ $f(x) = e^{\frac{2x}{x-1}}$
5. (10 points) Consider the function $f(x) = 3x^3 - 11x^2 + 8x + 4$.
   (a) Find the critical points.
   (b) Find the interval(s) where function is increasing interval(s) where it is decreasing.
   (c) Find the local maxima and minima.
   (d) Find the inflexion points.
   (e) State the interval(s) where f is concave up and the interval(s) where it is concave down.
   (f) Sketch the graph of the function.
6. Given the function $f(x) = e^x - 3x$.
   (a) (5 points) Apply the intermediate value theorem to show that there is a root of the equation $f(x) = 0$ between $a = 0$ and $b = 1$.
   (b) (5 points) Use Newton's method with an initial approximation $x_1 = .5$ to find two iterations x_2 and x_3. Give an approximation of the root of $f(x) = 0$ with two decimal places.
7. (5 points) Use linear approximation to estimate $\sqrt[3]{9}$.

8. (18 points) Find the antiderivatives in each case:

(a) $\int(x^2 - 5x^3 + 2)dx$ (a') $\int(x + \sec^2(x))dx$

(b) $\int x\cos(-4x + 7x^2)dx$ (b') $\int \frac{x^2}{2x^3-1}dx$

(c) $\int 3x^3(x^4 + 1)^{1/3}dx$ (c') $\int(x^{1/2} + 3x)dx$

9. (10 points) Find the area between the curves $y = x^2 - 2x - 3$ and the x-axis, when $0 \le x \le 4$.

10. (8 points) State the mean value theorem. Given the function $f(x) = x^2 - 5x + 3$, find a $c \in (0, 2)$ satisfying the conclusion of the mean value theorem.

11. (5 points) Use implicit differentiation to find $\frac{dy}{dx}$ if $x\sin(y) = y\sin(x)$.

12. (5 points) Given the function $f(x) = \frac{1}{x}$. Use 4 rectangles to approximate the area under the curve and between $x = 2$ and $x = 3$. Compare with exact value of the area $\int_1^2 1/x dx$.