MATH 46 ABSTRACT ALGEBRA. SECOND TEST. FALL 2011

1. Explain with your words the following concepts:
   (a) What does it means for a subset $H \subset G$, to be a subgroup $H \prec G$?
   (b) What is a cyclic subgroup?
   (c) What is a subgroup generated by a set of elements in a group G?
   (d) What is the order of an element $a \in G$?

2. Find the subgroup of $(\mathbb{Z}, +)$ generated by the elements 54 and 81.

3. Describe all the elements of the cyclic subgroup of $(Gl(2, \mathbb{R}), *)$ generated by

   (a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\quad$ (b) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

4. Let S be a subset (not necessarily a subgroup) inside a group G.
   Show that $H_S = \{x \in G \quad | \quad xs = sx \quad \forall s \in S\}$ is a subgroup of G.

5. Find the subgroup of S_5 generated by the element $\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix}$.

6. Find the group of symmetries on the regular hexagon. Give an example of a non-commutative subgroup of S_6 of order 12.

7. Express the following element of S_8: $\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 7 & 4 & 3 & 6 & 1 & 2 & 5 \end{pmatrix}$ as product of transpositions. Is the permutation ρ even or odd?

8. State the theorem of Lagrange. Prove that a finite cyclic group whose order is a prime number, has no proper non-trivial subgroups.

9. State the fundamental theorem of finite abelian groups. Find all abelian groups of order 2012.

10. Fill up the following table (if possible) with examples of non isomorphic finite groups:

Cyclic Group	Abelian non Cyclic	Non Abelian

11. Fill up the following table (if possible) with examples of non isomorphic infinite groups:

Cyclic Group	Abelian non Cyclic	Non Abelian