Midterm Practice Math 31 Spring 2020

1. Given the polynomial function $f(x) = x^2 + 3x + 5$:
   (a) Find the derivative of the function $f(x)$ at $x_0 = -1$ **using the definition**.
   (b) Check your answer evaluating the derivative $f'(x)$ of $f(x)$ at $x_0 = -1$.

2. Find the tangent line to the graph of the function $f(x) = e^{x^2+2x}$ at the point $x_0 = 0$. **You can use the rules of derivatives, no need to go back to the definition**.

3. Find the limits if they exist. Otherwise explain your answer:
   (a) $\lim_{x\to -2} \frac{3x^2+x-2}{x^2-1}$
   (b) $\lim_{x\to 2} \frac{x^2-6x+8}{x^2-5x+6}$
   (c) $\lim_{x\to 4^+} \frac{-1}{x-4}$
   (d) $\lim_{x\to 4^-} \frac{-1}{x-4}$
   (e) $\lim_{x\to 4} \frac{-1}{x-4}$
   (f) $\lim_{x\to 0} \frac{\sin(3x^2)}{\sin(7x^2)}$
   (g) $\lim x \to 0\frac{x^2}{x^2+1}$
   (h) $\lim_{x\to 0} \frac{\sin(4x^4)}{\sin(5x^3)}$

4. Given the following function $f(x)$. Determine the domain of continuity, i.e. the real numbers x where f is continuous.

$$f(x) = \begin{cases} \dfrac{3x}{x-2} & \text{if } x \leq 0, \\ \dfrac{x^2-4}{x^2+1} & \text{if } 0 < x \leq 2, \\ \dfrac{x^2-4x+4}{x^2+x-6} & \text{if } x > 2 \end{cases}$$

5. Find the derivative of the following functions:
   (a) $f(x) = \dfrac{x^2-5}{x^2+3}$
   (b) $f(x) = \sqrt{3x-4}$
   (c) $f(x) = x^3 \sin(x)$
   (d) $f(x) = \cos(4x^2 - 8x + 1)$
   (e) $f(x) = \dfrac{-18}{x^2}$
   (f) $f(x) = e^{x^2-2x}$

6. Consider the function $f(x) = x^3 - 27x + 44$.
   (a) Find the zeroes (x-intercepts) of $f(x)$.

(b) Find the critical points. You are going to need later for the graph the y-coordinate for each of these x's.

(c) Find the interval(s) where function is increasing as well as the interval(s) where it is decreasing.

(d) Find the local maxima and minima.

(e) Sketch the graph of the function.

7. State the conditions that a function must satisfy to apply the mean value theorem. Given the function $f(x) = x^2 - 5x + 3$, explain why we can use the theorem for f in the interval $(0, 3)$. Find a $c \in (0, 3)$ satisfying the conclusion of the mean value theorem.

8. State the conditions that a function must satisfy to apply the intermediate value theorem. Use the intermediate value theorem to show that there is a root of the equation $e^{3x} = 21x$ in the interval $[0, 1]$. Make sure to state what function are you working with and why it satisfies the conditions.

9. Use implicit differentiation to find the equation of the tangent line to the graph of equation $x^3 + 8xy + y^3 = 10$ at the point $(1, 1)$.