

MATH 46 ABSTRACT ALGEBRA. FIRST TEST. FALL 2011

1. State in your own words the definition of group. Check whether or not the following examples are groups:
 - (a) \mathbb{R}^+ with the operation $a * b = \sqrt{a * b}$.
 - (b) $n \times n$ matrices with determinant $+1$ or -1 and matrix multiplication.
 - (c) \mathbb{R} with the operation $a * b = |a * b|$.
 - (d) \mathbb{Z}_7 with multiplication *mod* 7.
 - (e) $\{a + bi | a, b \in \mathbb{Z}\}$ under addition.
 - (f) $\{f \text{ continuous } | f : [0, 1] \mapsto [0, 1]\}$ under composition of functions.
2. Let $(G, *)$ be a group and $a \in G$. What does it mean for an element a' to be inverse of a ?
 - (a) Prof that there is a unique inverse for every element of the group.
 - (b) Proof that $b'a'$ is the inverse of ab .
3. Show that if $(a * b)^2 = a^2 * b^2$ for all $a, b \in G$, then the group G is Abelian.
4. Establish with your words the concept of group isomorphism. Give an example of a pair of isomorphic groups and a pair of groups that are not isomorphic.
5. Find two different non isomorphic groups with 6 elements and explain why are they are not isomorphic.
6. * Show that any group with 5 elements is abelian.