Let R be an associative ring with 1. Consider a homogeneous system in $k+1$ variables (with scalars written on the left). In every left unitary R-module M, its solution set is a subgroup of the additive group of M^{k+1}. Consider the map φ that assigns to every left R-module M the projection onto the last coordinate of that solution set, which is a subgroup of the additive group of M itself. It is easy to see that φ is in fact a functor from left R-modules to Abelian groups, $\varphi : R\text{-Mod} \to \text{Ab}$.

Most prominent examples are $s|x$ for a given $0 \neq s \in R$, which defines the projection onto the x-coordinate of the equation $sy = x$ (here the above $k = 1$), and $rx = 0$ (where $k = 0$—no projection involved).

I call φ `high` if, in every injective module E, it defines all of E, i.e., $\varphi(E) = E$. For instance, over a domain all injective modules are divisible, which means that all the above divisibility functors of the form $s|x$ are high. Long time (almost exactly 35 years) ago, I discovered this dichotomy: every such φ is either high or else `bounded`, by which I mean that some fixed nonzero ring element r annihilates $\varphi(M)$, i.e., $r\varphi(M) = 0$, uniformly in all M.

I had not given this much further thought until recently, when I discovered, in collaboration with A. Martsinkovsky, that the dual notion of `low` functor gives rise to a torsion theory, namely `injective torsion` as introduced by him and J. Russell (and presented by him recently in this very seminar). This leads to a dichotomy dual to the one above. I will present my findings about their interaction, in particular, some curious characterizations of domains, of one-sided Ore domains, and also of two-sided Ore domains.