Twelfth Power Moment of Dirichlet L-functions

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Power moments for the Riemann zeta function involve a study of the average value of $|\zeta(1/2+it)|^{2k}$. In the realm of Dirichlet L-functions, natural analogues are power moments in the q-aspect, where $|L(1/2,\chi)|^{2k}$ is averaged over primitive characters χ modulo q. The Lindelöf hypothesis states that each moment is $\ll_\varepsilon q^{1+\varepsilon}$, but understanding these moments becomes difficult rapidly as k increases. Power saving asymptotics are known unconditionally only up to the $k=2$ case due to recent work by Young. Recently, Nunes produced a strong bound on the twelfth moment for Dirichlet L-functions with smooth squarefree moduli, an adaptation of the analogous result of Heath-Brown for the Riemann zeta function in the t-aspect. We will discuss how the framework of this proof may be applied to Dirichlet L-functions with prime power conductor and explore the rather different methods of evaluating and estimating exponential sums that arise in this setting.