Math 33, Practice Final Exam

This practice final is to give you an idea of what to expect and help us review. On the final you will be asked to do any 14 of the 17 questions in 2 hours and 50 minutes. They are worth 7 points each and to get these points you must show clearly all your working out and reasoning. You may use a scientific calculator. Phones must be put away. Use the restroom before the exam, not during it. (If you do more than 14 questions then I will take your best 14.)

---

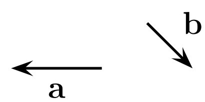
(1) *[Section 10.6] The polar equations*

$$r = \frac{ed}{1 \pm e\cos\theta}, \qquad r = \frac{ed}{1 \pm e\sin\theta}$$

give conic sections with eccentricity e*, a focus at the origin, and directrix at* $x = \pm d$ *or* $y = \pm d$*.* Give the eccentricity of and then sketch the polar curve:

$$r = \frac{4}{3 + 2\cos\theta}$$

(2) *[Section 12.2]* For these vectors



just use this picture to accurately draw the new vectors:

- **(a)** $\mathbf{a} + \mathbf{b}$
- **(b)** $3\mathbf{b}$
- **(c)** $-\mathbf{a}$
- **(d)** $\mathbf{a} - \mathbf{b}$

(3) *[Sections 13.1, 13.2]* Let

$$\mathbf{r}(t) = e^{3t}\mathbf{i} + (-1 + \sin(2t))\mathbf{j} + \ln(t+1)\mathbf{k}$$

and find the parametric equations of the tangent line to this space curve when $t = 0$

(4) *[Section 13.3]* Use the arc length formula

$$L = \int_a^b |\mathbf{r}'(t)|\, dt$$

to find the length of the parametric curve

$$\mathbf{r}(t) = \langle 2t, t^2, \tfrac{1}{3}t^3 \rangle, \qquad 0 \leqslant t \leqslant 1.$$

(5) *[Section 14.4]* Find the equation of the tangent plane to $z = \ln(x - 2y)$ at the point $(3, 1, 0)$.

(6) *[Section 14.5]* With

$$z = \ln(2x + 3y), \quad x = s\cos t, \quad y = t\sin s$$

use the chain rule to find: $\partial z/\partial s$

(7) *[Section 14.7] Recall that a critical point (a, b) of $f(x, y)$ can be classified using these formulas. Let $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}$. Then $D > 0$, $f_{xx} > 0$ means a local min; $D > 0$, $f_{xx} < 0$ means a local max; $D < 0$ means a saddle.*

Find and classify all the critical points of:

$$f(x, y) = xy - 2x - 2y - x^2 - y^2$$

(8) *[Section 14.8]* Use Lagrange multipliers to find the extreme values of $f(x, y) = x^2 - y^2$ subject to the constraint $x^2 + y^2 = 1$.

(9) *[Section 15.2]* Let W be the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$. Sketch this region and then evaluate the double integral: $\displaystyle\iint_W \frac{y}{x^2 + 1}\, dA$

(10) *[Section 15.5] Area formula. The function $f(x, y)$ has a graph $z = f(x, y)$ that makes a surface. The area of the part S of this surface that lies over a region D in the xy-plane is given by*

$$A(S) = \iint_D \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$$

Find the area of the part of the plane $6x + 4y + 2z = 1$ that lies inside the cylinder $x^2 + y^2 = 25$.

(11) *[Section 15.6]* Draw the tetrahedron with vertices at the points

$$(0, 0, 0), \quad (2, 0, 0), \quad (0, 2, 0), \quad (0, 0, 2).$$

Its sides are the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 2$. Now use a triple integral to find its volume.

(12) *[Section 15.9]* Let S be the unit sphere $\{(u, v, w) \mid u^2 + v^2 + w^2 = 1\}$ and recall that it contains a volume of $4\pi/3$. Under the change of variables $x = au$, $y = bv$, $z = cw$ for constants a, b, c we obtain an ellipsoid in xyz space:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(a) Find the Jacobian $\dfrac{\partial(x, y, z)}{\partial(u, v, w)}$

(b) Compute the volume inside this ellipsoid by using a triple integral and the given change of variables.

(13) *[Section 16.2]* An object moves in a straight line C from $(0,0,0)$ to $(4,4,4)$. It is moving in the force field $\mathbf{F} = \langle -x, -y, -z \rangle$. Calculate the work done by $\mathbf{F}$ on this object. This is

$$W = \int_C \mathbf{F} \cdot d\mathbf{r}$$

(14) *[Section 16.3]* Determine whether or not the vector field

$$\mathbf{F}(x,y) = (2xy+1)\,\mathbf{i} + (x^2+2y)\,\mathbf{j}$$

is conservative. If it is find an f so that $\nabla f = \mathbf{F}$.

(15) *[Section 16.4] Green's Theorem: Let D be a region with positively oriented boundary curve C. Then*

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA.$$

Use Green's Theorem to evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where

$$\mathbf{F}(x,y) = y^2 \cos x\ \mathbf{i} + (x^2 + 2y \sin x)\,\mathbf{j}$$

and C is the triangle from $(0,0)$ to $(2,6)$ to $(2,0)$ to $(0,0)$.

(16) *[Section 16.8] Stokes' Theorem: Let S be an oriented surface in $\mathbb{R}^3$ with positively oriented boundary ∂S. Let $\mathbf{F}$ be a vector field. Then*

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}.$$

Let $\mathbf{F}(x,y,z) = \langle 2y, xz, x+y \rangle$ and let C be the curve where the plane $z = y+2$ meets the cylinder $x^2+y^2 = 1$, (with C oriented counter clockwise when viewed from above). Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

by first using Stokes' Theorem to convert it to a surface integral.

(17) *[Section 16.9] The Divergence Theorem: Let E be a solid region with S its boundary surface, oriented outwards. Let $\mathbf{F}$ be a vector field. Then*

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \operatorname{div} \mathbf{F}\, dV.$$

Use the Divergence Theorem to calculate the flux of

$$\mathbf{F}(x,y,z) = x^2y\mathbf{i} + xy^2\mathbf{j} + 2xyz\mathbf{k}$$

across S where S is the surface of the box $E = [0,1] \times [0,2] \times [0,3]$.
