

Mth 33, Homework 8 on sections 15.2, 15.3, 15.4

Due by Tue, Mar 31.

Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points.

Section 15.2 Double Integrals over General Regions

(1) Evaluate the iterated integral: $\int_0^2 \int_1^{y^2} x^2 y \, dx dy$

(2) Let D be the region between the line $y = x$ and the parabola $y = 3x - x^2$. So

$$D = \{(x, y) \mid 0 \leq x \leq 2, x \leq y \leq 3x - x^2\}.$$

Then evaluate the double integral: $\iint_D 4y \, dA$

(3) Let R be the region bounded by the lines $y = 0$, $x = 1$ and $y = x$. Sketch this region and then find: $\iint_R x^2 e^{xy} \, dA$

(4) Let W be the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$. Sketch this region and then find: $\iint_W \frac{y}{x^2 + 1} \, dA$

(5) Let T be the triangle in the xy -plane with vertices $(0, 1)$, $(1, 2)$ and $(2, 1)$. Show that the volume under the surface $z = 2xy$ and above T is $8/3$.

Section 15.3 Double Integrals in Polar Coordinates

(6) Let R be the region that is outside $x^2 + y^2 = 1$, inside $x^2 + y^2 = 9$ and above the x -axis. Sketch R and write $\iint_R f(x, y) \, dA$ as an iterated integral in polar coordinates.

(7) Use your work in the previous question to find the area of that region R .

(8) Let D be the region in the first quadrant between the lines $x = 0$, $y = x$ and inside the circle $x^2 + y^2 = 4$. Evaluate:

$$\iint_D (2x - y) \, dA$$

(9) Compute $\iint_D \cos \sqrt{x^2 + y^2} \, dA$ when D is the disk of radius 3 centered at the origin. (This needs integration by parts.)

(10) Use a double integral to find the area inside the polar spiral $r = \theta^{3/2}$ with $0 \leq \theta \leq 2\pi$.

Section 15.4 Applications of double integrals

Formulas. If a lamina in region D has density $\rho(x, y)$ at the point (x, y) then its total mass m and its center of mass (\bar{x}, \bar{y}) are given by

$$m = \iint_D \rho(x, y) dA, \quad \bar{x} = \frac{1}{m} \iint_D x\rho(x, y) dA, \quad \bar{y} = \frac{1}{m} \iint_D y\rho(x, y) dA.$$

(11) A rectangular lamina $D = [0, 2] \times [0, 1]$ has uniform density $\rho(x, y) = 3$.

(a) Use the mass formula to find its mass m .

(b) Use the formulas to find its center of mass (\bar{x}, \bar{y}) .

(c) Draw D and plot the center of mass. Does your answer make sense?

(12) A square lamina $[0, 1] \times [0, 1]$ has density $\rho(x, y) = xy$. Find its mass and its center of mass.

(13) Let D be the part of the unit disk in the first quadrant:

$$D = \{(x, y) \mid x^2 + y^2 \leq 1, x \geq 0, y \geq 0\}.$$

Find the center of mass of this lamina if its density at each point is proportional to the square of the distance from the origin.

(Hint: use polar coordinates. By symmetry it must be true that $\bar{x} = \bar{y}$, so you only need to compute \bar{x} .)

If you are stuck on a question:

- Ask me about it after class.
- Come to my office hours: Mon 4:30 - 5:30, Wed 4:30 - 5:30 in CP 317.
- Go to the Math Tutorial Lab in person in CP 303 or online.