Mth 33, Homework 7 on sections 14.7, 14.8, 15.1

Due by Wed, Mar 25.

Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points.

Section 14.7 Maximum and Minimum Values

(1) Suppose $(1, 3)$ is a critical point for $f(x, y)$.

(a) What does being a critical point mean?

(b) If $f_{xx}(1, 3) = 5$, $f_{xy}(1, 3) = -2$, $f_{yy}(1, 3) = 1$ then what kind of critical point is it?

(2) Find and classify all the critical points of

$$g(x, y) = xy - 2x - 2y - x^2 - y^2$$

(3) Show that the critical points of $h(x, y) = (x - y)(1 - xy)$ are $(1, 1)$ and $(-1, -1)$. Then classify them.

(4) Find the absolute maximum and absolute minimum of $f(x, y) = x^2 + xy - y$ on the rectangle $[0, 2] \times [-3, 0]$. This rectangle notation means $0 \leqslant x \leqslant 2$ and $-3 \leqslant y \leqslant 0$.

(Check the values of f at all critical points inside the rectangle and on the four sides of the rectangle.)

Section 14.8 Lagrange Multipliers

(5) Use Lagrange multipliers to find the extreme values of $f(x, y) = x^2 - y^2$ subject to the constraint $x^2 + y^2 = 1$.

(6) Find the extreme values of xe^y subject to the constraint $x^2 + y^2 = 2$. You can use the following steps:

(a) Write down the three equations from the method of Lagrange multipliers. We want to find x, y, λ that solve all three.

(b) Explain why $\lambda = 0$ is not possible and, after substituting, we must have $x^2 = y$.

(c) Use this in $x^2 + y^2 = 2$ to find all possibilities for x and y.

(d) Use these possibilities to show the maximum is e and the minimum is $-e$.

(7) Find the extreme values of $2x + 2y + z$ subject to the constraint $x^2 + y^2 + z^2 = 9$.

(8) Find the extreme values of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the two constraints:

$$x - y = 1, \qquad y^2 - z^2 = 1$$

Section 15.1 Double Integrals over Rectangles

(9) Compute decimal approximations to the double integral $\iint_R xy^2 \, dA$ for the rectangle $R = [1, 3] \times [0, 4]$ using the following double Riemann sums with $m = n = 2$, so breaking R into 4 smaller rectangles:

(a) using upper right sample points,

(b) using midpoints.

(10) Compute

$$\iint_R 2 \, dA, \qquad R = [0, 2] \times [0, 3]$$

by finding the volume of the corresponding solid. Sketch this solid using $x\ y\ z$ coordinates.

(11) Compute

$$\iint_R \sqrt{4 - x^2} \, dA, \qquad R = [0, 2] \times [0, 3]$$

by finding the volume of the solid. Sketch this solid using $x\ y\ z$ coordinates.

(It is part of a cylinder.)

(12) Calculate the iterated integral

$$\int_1^4 \int_0^2 (6x^2 y - 2x) \, dy dx$$

(13) Calculate the double integral $\iint_R xy^2 \, dA$ for $R = [1, 3] \times [0, 4]$ exactly by writing it as an iterated integral. Does your answer match the approximations in Question 9?

(14) Find

$$\iint_R ye^{-xy} \, dA, \qquad R = [0, 1] \times [0, 2]$$

(Do the partial integration with respect to x first. Use the substitution method.)

If you are stuck on a question:

- Ask me about it after class.
- Come to my office hours: Mon 4:30 - 5:30, Wed 4:30 - 5:30 in CP 317.
- Go to the Math Tutorial Lab in person in CP 303 or online.