Mth 33, Homework 6 on section 14.6

Due by Wed, Mar 18.

Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points.

Section 14.6 Directional Derivatives and the Gradient Vector

(1) For $f(x, y) = x^3 e^y$

- **(a)** Find ∇f. This is the gradient of f.
- **(b)** Find $\nabla f(2, 0)$. This is the gradient of f evaluated at the point $(2, 0)$.
- **(c)** Find $D_{\mathbf{u}} f(2, 0)$ for $\mathbf{u} = \langle 3/5, -4/5 \rangle$. This is the directional derivative of f at the point $(2, 0)$ in the direction of the unit vector $\mathbf{u}$.

(2) Find ∇g when

$$g(x, y, z) = \ln(4x + y) - 8 \arcsin(x + z)$$

(3) For $f(x, y, z) = x^2 y + y^2 z$

- **(a)** Find ∇f.
- **(b)** Find ∇f at $P(1, 2, 3)$.
- **(c)** Find $D_{\mathbf{u}} f$ at $P(1, 2, 3)$ in the direction from $P(1, 2, 3)$ to $Q(3, 1, 5)$.

(4) Think of the graph of $z = f(x, y)$ as a surface, with hills and valleys. If you are standing on this surface and looking around, how can you tell which direction the gradient is pointing in? Explain in your own words.

(5) With $h(x, y) = 5xy^2$ find the maximum rate of change of h at $(3, -2)$ and give the direction of this maximum rate of change.

(The maximum rate of change is in the direction $\nabla h(3, -2)$ and equals $|\nabla h(3, -2)|$.)

(6) With $g(r, s, t) = \arctan(rst)$ find the maximum rate of change of g at $(1, 2, 1)$ and give the direction of this maximum rate of change.

If you are stuck on a question:

- Ask me about it after class.
- Come to my office hours: Mon 4:30 - 5:30, Wed 4:30 - 5:30 in CP 317.
- Go to the Math Tutorial Lab in person in CP 303 or online.