Mth 33, Homework 5 on sections 14.3 – 14.5
Due by Mon, Mar 16.

Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points.

Section 14.3 Partial Derivatives

(1) For $f(x, y) = 3x^4 - 6xy^3$ find these first partials:

- **(a)** f_x
- **(b)** $f_x(1, -1)$
- **(c)** f_y
- **(d)** $f_y(1, -1)$

(2) Find the first partials of $g(r, \theta) = \sin(r \cos \theta)$

(3) The equation

$$x^3 - y^2 + z^2 - 2z - 3 = 0$$

defines z implicitly as a function of x and y. Find: $\dfrac{\partial z}{\partial x}$

(4) Let $f(x, y) = \sin(x^2 y)$ and compute these first and second partials:

- **(a)** f_x
- **(b)** f_y
- **(c)** f_{xx}
- **(d)** f_{xy}
- **(e)** f_{yx}
- **(f)** f_{yy}
- **(g)** Do we get a confirmation of Clairaut's Theorem? Explain.

(5) If $V = \ln(r + s^2 + t^3)$ then find: $\dfrac{\partial^3 V}{\partial r \partial s \partial t}$

Section 14.4 Tangent Planes and Linear Approximations

(6) Find the equation of the tangent plane to $f(x, y) = 3x^2 + x - y^3$ at the point $(1, 1, 3)$.

(7) Find the equation of the tangent plane to $z = \ln(x - 2y)$ at the point $(3, 1, 0)$.

(8) Let $f(x, y) = x^2y^3$.

(a) Use a theorem to explain why $f(x, y)$ is differentiable.

(b) Find $L(x, y)$, the linear approximation to $f(x, y)$ at $(1, 1)$.

(c) Compare $L(0.95, 1.08)$ with $f(0.95, 1.08)$

(9) Verify the linear approximation at $(0, 0)$

$$\frac{y-1}{x+1} \approx x + y - 1$$

Section 14.5 The Chain Rule

(10) Suppose $z = xye^y$, $x = t^2$, $y = 5t$, Compute dz/dt in two ways as follows.

(a) Use the chain rule.

(b) Write z as a function of t explicitly by substituting for x and y. Then find dz/dt.

(Write your answer from part (a) just in terms of t and check it equals your answer from (b).)

(11) With

$$z = \ln(2x + 3y), \quad x = s\cos t, \quad y = t\sin s$$

use the chain rule to find $\partial z/\partial s$ and $\partial z/\partial t$.

(12) Suppose w is a function of x, y, z and also x, y, z are each functions of u and v. In other words

$$w = f(x, y, z), \quad x = x(u, v), \quad y = y(u, v), \quad z = z(u, v).$$

(a) Draw a tree diagram for these variables.

(b) Give the chain rule for $\partial w/\partial v$.

(13) When

$$z = xy + 2x - y, \quad x = rst, \quad y = r\sin(st)$$

use the chain rule to find $\partial z/\partial s$ at $r = 1$, $s = 0$, $t = 2$.

(14) Suppose $x^2 + y^3 + z^4 - 3xz + 3 = 0$. Find $\partial z/\partial x$ and $\partial z/\partial y$ using the following formulas. If $F(x, y, z) = 0$ then

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}.$$

If you are stuck on a question:

- Ask me about it after class.
- Come to my office hours: Mon 4:30 - 5:30, Wed 4:30 - 5:30 in CP 317.
- Go to the Math Tutorial Lab in person in CP 303 or online.