Mth 33, Homework 2 on sections 10.5, 10.6, 12.1

Due by Wed, Feb 11.

Try these questions yourself. Write all your working out and answers neatly by hand on your own notepaper and hand them to me next week. Please use lots of space and as many pages as you want, so I can include corrections or comments - otherwise I may ask you to redo it. You do not need to write the questions, but it is very important that you show clearly all the steps you had to do to get your answers.

Each question is worth 3 points.

Section 10.5 Conic Sections

Formulas you might need:

- A parabola with focus $(0, p)$ and directrix $y = -p$ has equation $x^2 = 4py$.
- A parabola with focus $(p, 0)$ and directrix $x = -p$ has equation $y^2 = 4px$.
- The equation
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a \geqslant b > 0$$
is an ellipse with foci $(\pm c, 0)$ for $c^2 = a^2 - b^2$, and vertices $(\pm a, 0)$.
- The equation
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \qquad a \geqslant b > 0$$
is an ellipse with foci $(0, \pm c)$ for $c^2 = a^2 - b^2$, and vertices $(0, \pm a)$.
- The equation
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
is a hyperbola with foci $(\pm c, 0)$ for $c^2 = a^2 + b^2$, vertices $(\pm a, 0)$ and asymptotes $y = \pm bx/a$.
- The equation
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$
is a hyperbola with foci $(0, \pm c)$ for $c^2 = a^2 + b^2$, vertices $(0, \pm a)$ and asymptotes $y = \pm ax/b$.

(1) The equation $y = x^2/8$ is the equation of a parabola. Find its focus and directrix and graph these along with the parabola.

(2) Give the equation of the parabola with vertex at $(0, 0)$ and directrix $x = 3/4$.

(3) For the ellipse with equation

$$\frac{x^2}{25} + \frac{y^2}{16} = 1,$$

(a) Find its two vertices.

(b) Find its two foci.

(c) Sketch this ellipse, showing its vertices and foci.

(4) For the hyperbola with equation

$$x^2 - \frac{y^2}{4} = 1,$$

(a) Find its two vertices.

(b) Find its two foci.

(c) Find its asymptote lines.

(d) Sketch this hyperbola, showing the results from parts (a), (b), (c).

(5) Identify which type of conic this is and find its foci:

$$3x^2 - 12 = -4y^2$$

Section 10.6 Conic Sections in Polar Coordinates

Formulas:

- The polar equations

$$r = \frac{ed}{1 \pm e\cos\theta}, \qquad r = \frac{ed}{1 \pm e\sin\theta}$$

  give conic sections with eccentricity e, a focus at the origin, and directrix at $x = \pm d$ or $y = \pm d$.

(6) Give the polar equations of

(a) A parabola with focus at the origin and directrix $x = 5$.

(b) A hyperbola with focus at the origin, eccentricity 6 and directrix $x = 2$.

(7) For this polar equation

$$r = \frac{4}{3 + 2\cos\theta}$$

(a) Find its eccentricity.

(b) Which type of conic section is it?

(c) Give the equation of its directrix.

(d) Sketch this conic and its directrix.

(8) For this polar equation

$$r = \frac{15}{2 + 3\cos\theta}$$

(a) Find its eccentricity.

(b) Which type of conic section is it?

(c) Give the polar coordinates of its vertices.

Section 12.1 Three-Dimensional Coordinate Systems

(9) Draw the x, y, z axes for $\mathbb{R}^3$ and plot these three points on them:

$$A(1, 4, 3), \quad B(0, 3, 0), \quad C(-1, 1, -4)$$

(10) For the point $P(4, 1, 2)$,

(a) Give its projection on the xy-plane.

(b) Give its projection on the xz-plane.

(c) Give its projection on the yz-plane.

(d) Plot all 4 of these points together.

(11) Find the distance between the points $(6, -3, 0)$ and $(2, 4, 5)$. Give your answer as a simplified radical and as a decimal.

(12) In your own words, describe the regions in $\mathbb{R}^3$ represented by each of these:

(a) $y = 2$

(b) $x^2 + y^2 \leqslant 4$

(c) $x^2 + y^2 + z^2 = 16$

Many of you are using AI to help with homework questions. You can paste them in and get step by step answers most of the time. If you are using AI, the smart way is:

- Study the relevant class notes and section in the textbook first.
- Try the homework questions yourself.
- Use AI help if you're having trouble.
- Make sure you understand the answers you're handing in - there will be similar questions on exams.

If you're still stuck on a question:

- Ask me about it after class.
- Come to my office hours: Mon 4:30 - 5:30, Wed 4:30 - 5:30 in CP 317.
- Go to the Math Tutorial Lab in person in CP 303 or online.