**Mth 33, Homework 12 on sections 16.5, 16.6, 16.7**

Due by Wed, May 13.

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Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points.

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### Section 16.5 Curl and Divergence

Formulas.

$$\operatorname{grad} f = \nabla f, \qquad \operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F}, \qquad \operatorname{div} \mathbf{F} = \nabla \cdot \mathbf{F}$$

**(1)** Define the vector field

$$\mathbf{F}(x, y, z) = \langle e^x, e^{xy}, e^z \rangle$$

and compute

**(a)** its divergence: $\nabla \cdot \mathbf{F}$

**(b)** its curl: $\nabla \times \mathbf{F}$.

**(2)** Let

$$\mathbf{G}(x, y, z) = y \sin z \, \mathbf{i} + x \sin z \, \mathbf{j} + xy \cos z \, \mathbf{k}.$$

Compute $\nabla \times \mathbf{G}$ and use it to decide if $\mathbf{G}$ is conservative or not.

**(3)** Let

$$\mathbf{F} = xe^y \, \mathbf{i} + e^{x+y} \, \mathbf{j} + xyz \, \mathbf{k}.$$

Evaluate the curl of $\mathbf{F}$ at the origin. Draw this vector. Then sketch and explain how points near the origin will rotate.

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### Section 16.6 Parametric Surfaces and Their Areas

**(4)** Let

$$\mathbf{r}(u, v) = \langle u + v, u^2 - v, u + v^2 \rangle$$

be a surface parameterized by $(u, v) \in \mathbb{R}^2$. Which of the points $P(3, -1, 5)$ and $Q(-1, 3, 4)$ are on this surface?

**(5)** Find an equation of the tangent plane to the parametric surface

$$\mathbf{r}(u, v) = \langle u + v, 3u^2, u - v \rangle$$

at $\mathbf{r}_0 = \langle 2, 3, 0 \rangle$.

(First find what $u$ and $v$ must be at this point. Then compute the partials $\mathbf{r}_u$, $\mathbf{r}_v$ there and find their cross product to obtain the normal vector $\mathbf{n}$ to the plane. The plane is then given by the equation $(\langle x, y, z \rangle - \mathbf{r}_0) \cdot \mathbf{n} = 0$.)

**(6)** Let $S$ be the surface parameterized by

$$\mathbf{r}(u,v) = 3\cos u\,\mathbf{i} + 3\sin u\,\mathbf{j} + v\,\mathbf{k} \quad \text{where} \quad 0 \leqslant u \leqslant \pi/2,\ 0 \leqslant v \leqslant 2.$$

Carefully sketch $S$ in the $xyz-$space.

**(7)** Find the surface area of $S$ from the previous question by using the formula

$$A(S) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v|\, dA.$$

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### Section 16.7 Surface Integrals

Formulas. For a surface $S$ parameterized by $\mathbf{r}(u,v)$ with $(u,v) \in D$, we have the surface integrals

$$\iint_S f\, dS = \iint_D f(\mathbf{r}(u,v))|\mathbf{r}_u \times \mathbf{r}_v|\, dA,$$

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_D \mathbf{F}(\mathbf{r}(u,v)) \cdot (\mathbf{r}_u \times \mathbf{r}_v)\, dA$$

**(8)** Let $S$ be the surface given by

$$\mathbf{r}(u,v) = u^2\,\mathbf{i} + u\sin v\,\mathbf{j} + u\cos v\,\mathbf{k} \quad \text{where} \quad 0 \leqslant u \leqslant 1,\ 0 \leqslant v \leqslant \pi/2.$$

Find:

$$\iint_S yz\, dS$$

**(9)** Let

$$\mathbf{F}(x,y,z) = \langle x, x^2, -y \rangle$$

and let $S$ be the part of the paraboloid $z = x^2 + 3y^2$ that lies above the square $0 \leqslant x \leqslant 1$, $0 \leqslant y \leqslant 1$ and is oriented downward. Evaluate:

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

**(10)** Let

$$\mathbf{F}(x,y,z) = xy\,\mathbf{i} + x\,\mathbf{j} + z\,\mathbf{k}$$

and let $H$ be the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geqslant 0$ oriented away from the origin. Show that the flux of $\mathbf{F}$ across $H$ is

$$\iint_H \mathbf{F} \cdot d\mathbf{S} = 2\pi/3.$$

(For this question use the spherical coordinates parameterization for $H$:

$$\mathbf{r}(\phi,\theta) = \langle \sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi \rangle, \qquad 0 \leqslant \phi \leqslant \pi/2, \quad 0 \leqslant \theta \leqslant 2\pi,$$

and that

$$\mathbf{r}_\phi \times \mathbf{r}_\theta = \langle \sin^2\phi\cos\theta, \sin^2\phi\sin\theta, \sin\phi\cos\phi \rangle$$

gives an outward normal.)

If you are stuck on a question:

- Ask me about it after class.
- Come to my office hours: Mon 4:30 - 5:30, Wed 4:30 - 5:30 in CP 317.
- Go to the Math Tutorial Lab in person in CP 303 or online.