Mth 33, Homework 1 on sections 10.1 – 10.4

Due by Wed, Feb 4.

Try these questions yourself. Write all your working out and answers neatly by hand on your own notepaper and hand them to me next week. Please use lots of space and as many pages as you want, so I can include corrections or comments - otherwise I may ask you to redo it. You do not need to write the questions, but it is very important that you show clearly all the steps you had to do to get your answers.

Each question is worth 3 points.

Section 10.1 Curves Defined by Parametric Equations

(1) Sketch the curve defined by these parametric equations

$$x = 2 - t^2, \quad y = t^3 + t, \quad -2 \leqslant t \leqslant 2.$$

For this, make a table of values for $t = -2, -1, 0, 1, 2$, plot these 5 points and join them into a curve. Your sketch should include the x and y axes with their numbers neatly marked.

(2) Eliminate t from the parametric equations

$$x = t^2 + 2, \quad y = t + 2$$

to get the Cartesian equation of this curve.

(3) **(a)** Draw the graph of $x = -\sin(\pi t)$ for $-1 \leqslant t \leqslant 1$.

(b) Draw the graph of $y = 1 - |t|$ for $-1 \leqslant t \leqslant 1$.

(c) Use parts (a) and (b) to sketch the graph of the parametric curve

$$x = -\sin(\pi t), \quad y = 1 - |t|, \quad -1 \leqslant t \leqslant 1.$$

Section 10.2 Calculus with Parametric Curves

(4) For the parametric equations

$$x = t + e^t, \quad y = 4t - \cos t$$

find dx/dt, dy/dt and dy/dx.

(5) For the curve in the last question, use your formula for dy/dx to find the equation of the tangent line to this curve at $t = 0$.

(6) The cycloid is given by

$$x = r(\theta - \sin\theta), \quad y = r(1 - \cos\theta).$$

(a) Find the slope of the tangent line to it when $\theta = \pi/2$.

(b) Sketch the graph of the cycloid for $0 \leqslant \theta \leqslant 2\pi$ (one arch) and include your tangent line from part (a).

(7) For the parametric curve

$$x = 1 - t^2, \quad y = t - t^3$$

(a) Show that

$$\frac{d^2y}{dx^2} = -\frac{3t^2 + 1}{4t^3}.$$

(b) For which values of t is this curve concave up and concave down?

(8) Find the length of this curve

$$x = e^t - t, \quad y = 4e^{t/2}, \quad 0 \leqslant t \leqslant 2,$$

using the arc length formula

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

Section 10.3 Polar Coordinates

(9) Plot these three points, given in polar coordinates, and find their Cartesian coordinates: $A(2, \pi/2),\ B(-1, \pi/4),\ C(2, -\pi/3)$

(10) Plot these three points, given in Cartesian coordinates, and find their polar coordinates: $P(2, 2),\ Q(-3, 3),\ R(-2, 0)$

(11) Sketch the graph of the polar curve: $r = 2 + \cos(\theta)$

(12) Sketch the graph of the polar curve: $r = \sin(2\theta)$

Section 10.4 Calculus in Polar Coordinates

(13) Use the area formula $A = \int_a^b \frac{1}{2} r^2\, d\theta$ to find the area of the polar curve:

$$r = 2\cos(\theta), \quad 0 \leqslant \theta \leqslant \pi$$

(14) Use the arc length formula $L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ to find the length of the polar spiral:

$$r = \theta^2, \quad 0 \leqslant \theta \leqslant 2\pi$$

Many of you are using AI to help with homework questions. You can paste them in and get step by step answers most of the time. If you are using AI, the smart way is:

- Study the relevant class notes and section in the textbook first.
- Try the homework questions yourself.
- Use AI help if you're having trouble.
- Make sure you understand the answers you're handing in - there will be similar questions on exams.

If you're still stuck on a question:

- Ask me about it after class.
- Come to my office hours: Mon 4:30 - 5:30, Wed 4:30 - 5:30 in CP 317.
- Go to the Math Tutorial Lab in person in CP 303 or online.