## 3.3 Power functions and polynomial functions

These notes are to help you to read through section 3.3 of the textbook.

Power functions are very simple – just

a real number times a power of x.

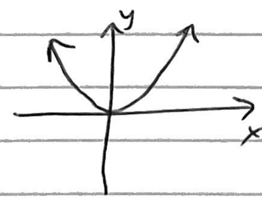
So things like $4x^5$ or $-\frac{3}{8}x^{-2}$

and we are mostly interested in the case when the power is a positive integer:

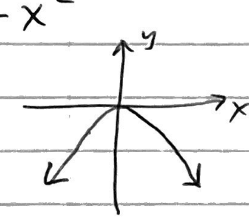
$$f(x) = kx^n \quad \text{for } n = 1, 2, 3, \ldots$$

Four key examples are

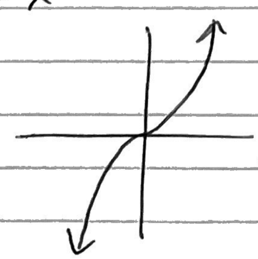
(A) $f(x) = x^2$



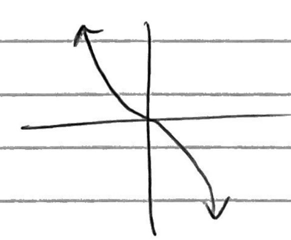

(B) $f(x) = -x^2$




(C) $f(x) = x^3$



(D) $f(x) = -x^3$



We see that as x gets bigger, eg $x=10$, $x=100$, $x=10000$ etc, the graphs in (A), (C) go up since $f(x)$ gets bigger too.

But in (B), (D) the graphs go down since $f(x)$ gets bigger in the negative direction

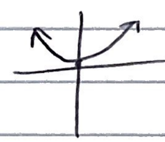
eg. for $f(x) = -x^3$ we have

$$f(10) = -10^3 = -1000$$
$$f(100) = -100^3 = -1000000$$

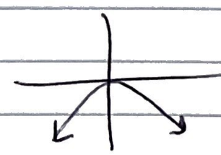
Similarly for $x$ going left with $x=-10$, $x=-100$ etc.

In general there are four cases for $f(x) = kx^n$

(A) $k>0$, $n$ even

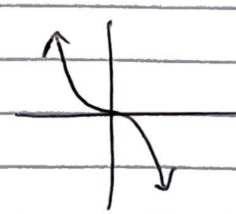


(B) $k<0$, $n$ even



(C) $k>0$, $n$ odd

(D) $k<0$, $n$ odd



If the graph goes up on the right we say $f(x) \to \infty$ as $x \to \infty$

If it goes down on the right
$f(x) \to -\infty$ as $x \to \infty$

Similarly if it goes up on the left
$f(x) \to \infty$ as $x \to -\infty$

and going down on the left
$f(x) \to -\infty$ as $x \to -\infty$.

This is called the <u>end</u> <u>behavior</u>.

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A <u>polynomial</u> <u>function</u> is a sum of these power functions, for example

$$f(x) = 7x^4 + 2x^3 - x^2 - 7x + 4.$$

The powers of $x$ must be positive integers (or zero if we write $4 = 4x^0$).

The numbers multiplying the powers of $x$ are called coefficients and can be any real numbers.

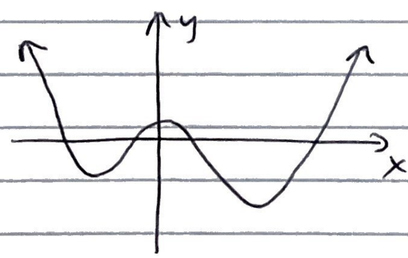
We usually write polynomials with the highest powers on the left, going down to the right.
The highest power in a polynomial is called its <u>degree</u>. The term containing

it is the leading term and the coefficient there is the leading coefficient.

So in our example:

- degree of f is 4
- leading term of f is $7x^4$
- leading coefficient of f is 7.

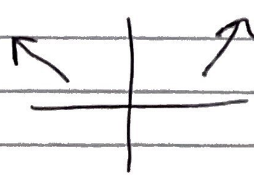
The graph of this f(x) is more complicated than the power functions we saw:




$y = f(x) = 7x^4 - 2x^3 - x^2 - 7x + 4$

A key idea is that it must have the same end behavior as its leading term. This is because the leading term becomes much bigger than the other terms as x gets bigger in the positive or negative directions.

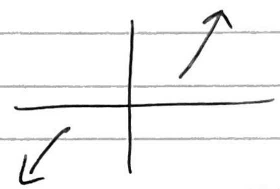
In our example, the end behavior of

$7x^4$ is  (case Ⓐ)

# 3.4 Graphs of polynomial functions

Example 1 find the end behavior of the polynomial function $f(x) = 4x^3 - 100x^2 + 11$

Solution. The leading term $4x^3$ gives the end behavior. With $4 > 0$ and 3 odd we're in case c and get



ie. $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$.

This helps us to graph $f(x)$ but we need more information. Finding where the graph must cross the x and y axes is our next step.

The y-intercept is easy — its at $f(0)$. To find the x-intercepts we need to solve $f(x) = 0$ and do this by factoring.

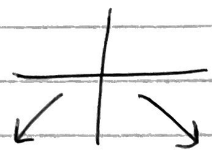
Example 2 For the polynomial $g(x) = -2(x+1)(x-3)$

find its end behavior, intercepts and graph.

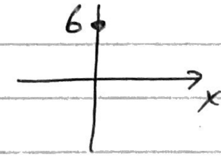
Solution. Multiplying the x terms we see the leading term of g is $-2x^2$

(altogether $g(x) = -2x^2 + 4x + 6$).

This is case B and the end behavior is



The y-intercept is $g(0) = 6$

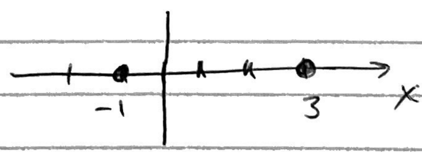



For the x-intercepts solve

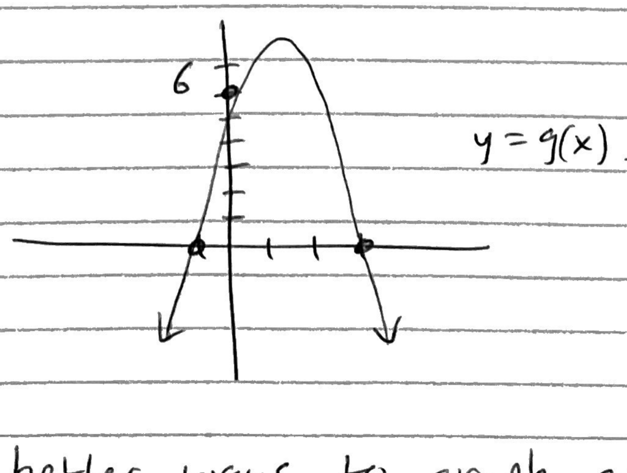
$$g(x) = -2(x+1)(x-3) = 0$$

$$x+1 = 0 \quad \text{or} \quad x-3 = 0$$

$$x = -1 \quad \text{or} \quad x = 3$$



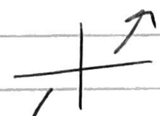

Putting all this together




(we saw better ways to graph quadratic functions in section 3.2. But the ideas here work for any degree polynomials.)

Example 3 Find the end behaviour, intercepts and graph $f(x) = 2x^3 + 2x^2 - 4x$.

Solution. The leading term $2x^3$ gives

case c behaviour 

The y-intercept is $f(0) = 0$.

We factor $f(x)$ to get the x-intercepts

$$f(x) = 2x^3 + 2x^2 - 4x$$

$$= \underbrace{2x}_{\text{GCF}}\underbrace{(x^2 + x - 2)}_{(x+\ )(x+\ )}$$

trinomial

$(\ )(\ ) = -2$

$(\ ) + (\ ) = 1$

need the numbers 2, −1 here

so $f(x)$ factors completely as

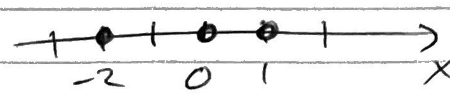
$$f(x) = 2x(x+2)(x-1)$$

Next solve

$$0 = f(x) = 2x(x+2)(x-1)$$

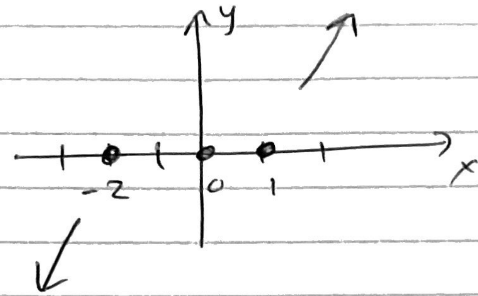
get $2x = 0$ or $x + 2 = 0$ or $x - 1 = 0$

$x = 0$ $x = -2$ $x = 1$

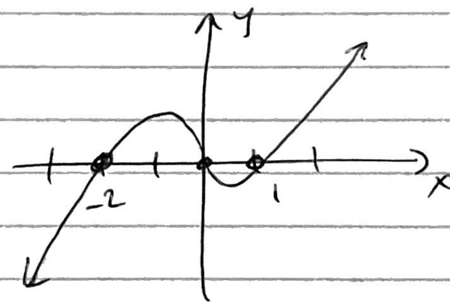



intercepts

So have




Polynomials have graphs that are smooth curves. Since we know the only points the graph can cross the axes, it must look like

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Review factoring in Chapter 6 of the Mth 28 textbook:

GCF: $3x^4 - 6x^2 \longrightarrow 3x^2(x^2-2)$

difference of squares $4x^2 - 9 \longrightarrow (2x+3)(2x-3)$

easy trinomial $x^2-4x-5 \longrightarrow (x+1)(x-5)$

harder trinomial $2x^2+7x+3 \longrightarrow (2x+1)(x-3)$

ac-method

Also factoring by grouping.