Mth 30, Homework 5 on sections 3.5, 3.6, 3.7

Due by Wed, Mar 12 or the following class.

Section 3.5 Dividing Polynomials

(1) Use long division (not synthetic division) to divide $2x^3 - 5x^2 + 5x - 3$ by $2x + 1$. Identify the quotient and the remainder.

(Hint: You should get a remainder of -7.)

(2) Divide $4x^3 + 10x^2 - 5x - 6$ by $x + 3$. Give the quotient and the remainder.

(Synthetic division is recommended - you should have $k = -3$.)

(3) Divide $f(x) = 3x^4 - 6x^3 - 5x + 10$ by $x - 2$. Is $x - 2$ a factor of $f(x)$?

(If using synthetic division, add a 0 for the missing power of x.)

(4) Let $g(x) = 2x^3 - 9x^2 - 3x + 8$. Evaluate $g(5)$ in two ways: first by substituting 5 into the formula, and secondly using synthetic division (with $k = 5$) to get the answer as a remainder. Make sure you get the same answer both ways!

(This is using the Remainder Theorem which says that if you divide $g(x)$ by $x - k$ then the remainder is $g(k)$.)

(5) Factor the polynomial $2x^3 + 7x^2 - 46x + 21$ completely by using that one factor is $x - 3$.

(Hint: Use synthetic division and then factor the quotient.)

Section 3.6 Zeros of Polynomials

(6) Suppose you know that $f(13) = 0$ for a certain polynomial $f(x)$. Can you say anything about the factors of $f(x)$? (Remember the Factor Theorem.)

(7) List the possible rational zeros of $3x^5 + 17x^4 - 19x + 4$ according to the theorem – no need to check if any are actual zeros.

(8) For the polynomial $f(x) = 2x^3 + x^2 - 7x - 6$,

- **(a)** List all possible rational zeros. (You should find 12 possibilities.)
- **(b)** Start testing to find one that is an actual zero by using synthetic division and looking for zero remainders.
- **(c)** When you find an actual zero $x = k$, use the quotient and $(x - k)$ to factor $f(x)$. Then factor the quotient (it might need the ac method).
- **(d)** Use the complete factorization of $f(x)$ to give all of its zeros, by the Factor Theorem.

(9) For the polynomial $f(x) = 2x^3 + 7x^2 - 5x - 4$,

(a) List all possible rational zeros.

(b) Find all the actual zeros of $f(x)$ by the same method as in the last question.

Section 3.7 Rational Functions

(10) Decide if these rational functions have horizontal asymptotes. If they do, give the equation of the horizontal asymptote line (it will be $y =$ a number). No need to graph these functions.

(a) $f(x) = \dfrac{x^3}{x^2+4}$ **(b)** $g(x) = \dfrac{5x}{x^2+4}$ **(c)** $h(x) = \dfrac{5x^3}{x^3+4}$

(Hint: the way to find horizontal asymptotes is to first compare the degrees of top and bottom. There are three possibilities...)

(11) Let $f(x)$ be the rational function

$$f(x) = \frac{x^2 - 1}{x^3 + 9x^2 + 14x}$$

and find

(a) its domain,

(b) the equations of the vertical asymptote lines,

(c) the equation of the horizontal asymptote line.

(Hint: Factor the bottom and see where it is zero to help answer parts (a) and (b). Remember that the equations of vertical lines are $x =$ number, and horizontal lines are $y =$ number.)

(12) For the rational function

$$g(x) = \frac{x-2}{x+1}$$

find its x and y intercepts. Find its vertical and horizontal asymptotes. With this information sketch the graph, using a table of values to find more points if needed.

(Remember, finding where the top is zero gives the x-intercepts, and finding where the bottom is zero gives the vertical asymptotes.)

(13) For the rational function

$$h(x) = \frac{x^2 - 3x - 4}{x^2 - x - 6}$$

find its x and y intercepts. Find its vertical and horizontal asymptotes. With this information sketch the graph, using a table of values to find more points if needed.

(14) Using the same steps as in the last question, carefully sketch the graph of:

$$f(x) = \frac{(2x-1)(x+3)}{(x+1)(x-3)}$$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes or section in the textbook.
- Ask me about it after class.
- Come to my office hours: Mon 2:00 - 3:00, Wed 2:00 - 3:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.