Mth 30, Homework 4 on sections 3.2, 3.3, 3.4

Due by Wed, Mar 5.

Section 3.2 Quadratic Functions

(1) For the quadratic function $f(x) = 2x^2 + 8x - 10$

- **(a)** Give the coordinates (h, k) of the vertex of this parabola.
  (Use the formulas $h = -b/2a$ and $k = f(h)$ to do this.)
- **(b)** Write $f(x)$ in the standard (vertex) form $f(x) = a(x - h)^2 + k$.
- **(c)** Give the equation of the axis of symmetry. (Since it is a vertical line, your answer should be $x =$ a number.)

(2) Let $f(x) = -x^2 - 2x + 3$

- **(a)** Find the vertex.
- **(b)** Find the y intercept.
- **(c)** Find the two x intercepts.
  (Here you want to solve $f(x) = -x^2 - 2x + 3 = 0$. To make that easier to solve, multiply both sides by -1 to get $x^2 + 2x - 3 = 0$.)
- **(d)** Use parts (a), (b), and (c) to carefully sketch the graph of $f(x)$.

(3) For the function in the previous question:

- **(a)** Give its domain.
- **(b)** Give its range.
- **(c)** Where is it increasing and decreasing?
- **(d)** Does it have a local maximum or minimum?

(4) Find the x intercepts of the graph of $g(x) = x^2 - 6x - 1$ using these steps:

- **(a)** We want to solve $0 = x^2 - 6x - 1$ but the right side does not factor. Instead first write this side in standard (vertex) form to get $0 = a(x - h)^2 + k$.
- **(b)** Put the square on one side: $(x - h)^2 = -k/a$
- **(c)** That means $x - h$ is plus or minus the squareroot of the right side. Then show that the x intercepts are $x = 3 - \sqrt{10}$ and $x = 3 + \sqrt{10}$. (So the graph crosses the x axis at approximately -0.16 and 6.16.)

(5) Find the two real numbers that have the biggest product if their sum is 12. Use these steps:

- **(a)** To understand what the question is asking you could try some numbers: for example $1 \cdot 11 = 11$ but $3 \cdot 9 = 27$ is bigger.

(b) Call the two numbers x and y. Then write the equation that means "their sum is 12" and then solve for y.

(c) Write the product xy as a function $f(x)$ that only depends on x.

(d) The x we want is the x coordinate of the vertex of the graph of $f(x)$ (since that is where the local maximum is).

(e) Give the two numbers x and y that solve the problem.

Section 3.3 Power Functions

(6) For the power function $f(x) = -2x^3$,

(a) Compute $f(10)$ and $f(100)$, showing the behavior going right.

(b) Compute $f(-10)$ and $f(-100)$, showing the behavior going left.

(7) For the same $f(x) = -2x^3$, fill in the blanks describing its end behavior:

(a) As $x \to \infty$, $f(x) \to$ ___.

(b) As $x \to -\infty$, $f(x) \to$ ___.

(Hint: your answers here should be ∞ or $-\infty$ and should match the previous question.)

(8) Give the end behavior of $g(x) = 12x^4 - 4x + 7$ by drawing the arrows that show the direction of the graph going left and right.

Section 3.4 Graphs of Polynomial Functions

(9) Find the x and y intercepts and end behavior of $f(x) = -2x(x-2)(x+5)$ and use this information to carefully sketch its graph. Make sure to label and number the axes.

(Hint: there are three x intercepts.)

(10) Find the x intercepts of $h(x) = 2x^4 - 8x^3 + 6x^2$

(Hint: to factor $h(x)$ take out the gcd first)

(11) Find the zeros of $f(x) = x^3 + 2x^2 - 9x - 18$

(Hint: The zeros are just another name for the x intercepts. Factor by grouping.)

(12) For $g(x) = (2x+1)^3(9x^2 - 6x + 1)$ use factoring to explain why its zeros are just $-1/2$ and $1/3$ and give their multiplicities.

(13) Sketch the graph of $h(x) = (x+3)^2(x-2)$ after finding its end behavior, intercepts and multiplicities of zeros. Make sure to label and number the axes.

(Remember that if the multiplicity is even then the graph does not cross the x-axis there.)

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes or section in the textbook.
- Ask me about it after class.
- Come to my office hours: Mon 2:00 - 3:00, Wed 2:00 - 3:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.