Mth 30, Homework 11 on sections 7.1, 7.2, 7.5
Due by Wed, May 7.

Section 7.1 Trigonometric Identities

(1) Fill in the following blanks describing how each step works. For example it could be "by the Pythagorean identity", "adding fractions", "by definition" or "distributing".

$$\begin{aligned}
\frac{\cot x + \tan x}{\sec x} &= \frac{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}{\frac{1}{\cos x}} && \underline{\hspace{4cm}} \\
&= \cos x \left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} \right) && \underline{\hspace{4cm}} \\
&= \frac{\cos^2 x + \sin^2 x}{\sin x} && \underline{\hspace{4cm}} \\
&= \frac{1}{\sin x} && \underline{\hspace{4cm}} \\
&= \csc x. && \underline{\hspace{4cm}}
\end{aligned}$$

The above steps have verified the identity $\dfrac{\cot x + \tan x}{\sec x} = \csc x$.

(2) Simplify

$$\frac{3\sin^2 x + 4\cos^2 x - 3}{\cos x}$$

to a single trigonometric function. Go step by step as in question 1 and don't forget to write the equality symbol "=" at each step, to show what is equal.

(3) Verify the identity:

$$\sin(-x)\tan x + \sec(-x) = \cos x$$

(Remember that $\cos$ and $\sec$ are even functions and the other trig functions are odd.)

(4) Verify the identity:

$$\cot^2\theta - \csc^2\theta = -1$$

(5) Verify the identity:

$$2 = \frac{(\cos\theta + 1)^2 + (\sin\theta + 1)^2 - 3}{\cos\theta + \sin\theta}$$

Section 7.2 Sum and Difference Identities

Use these sum identities in these three questions:

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$
$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$

(6) Find the exact value of: $\cos(105°)$

(Hint: Write $105°$ as a sum of special angles.)

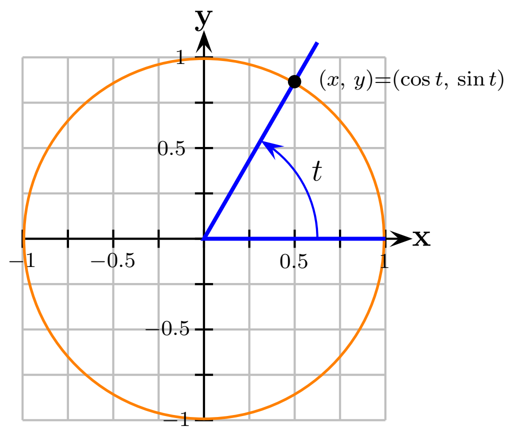
(7) Write $\sin(x - \pi/4)$ in terms of $\sin x$ and $\cos x$.

(8) Verify the identity:

$$\frac{\sin(x+y)}{\sin x \cdot \sin y} = \cot x + \cot y$$

Section 7.5 Solving Trigonometric Equations

The unit circle here is helpful for solving trigonometric equations. Also remember the values of $\cos$ and $\sin$ at the special angles. All angles are given with radians here.



(9) Find all solutions to: $\sin t = 1$

(Hint: There are infinitely many – find radian angles that have $y = 1$ on the unit circle above. The answer is "$t = ?? + 2\pi k$ for all integers k".)

(10) Solve $3\sin t = 1 + \sin t$ exactly for t in $[0, 2\pi)$.

(Hint: There are two solutions. On the unit circle you will be looking for two points with y coordinate $1/2$ and see which angles they correspond to.)

(11) Solve $1 + \cos t = 0$ exactly for t in $[0, 6\pi)$. (Three solutions.)

(12) Solve $\sin^2 t = 3/4$ exactly for t in $[0, 2\pi)$.

(13) Solve $2\cos(2\theta) = 1$ exactly for θ in $[0, 2\pi)$. (Now four solutions.)

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes or section in the textbook.
- Ask me about it after class.
- Come to my office hours: Mon 2:00 - 3:00, Wed 2:00 - 3:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.