MTH 30, Midterm. [9:00 - 10:50 am] Name (first, last): Solutions

Put your phone away while taking this test. You may use your own calculator or mine only. Do any working out in the space provided. Questions are worth 8 points each and you must show your work and explain your answers to get full credit.

Q1. For the functions $f(x) = 4x^2 - x$ and $g(x) = 2x - 3$ find the composition $f(g(x))$

$$f(g(x)) = f(2x-3)$$
$$= 4(2x-3)^2 - (2x-3)$$
$$= 4(4x^2 - 12x + 9) - 2x + 3$$
$$= 16x^2 - 48x + 36 - 2x + 3$$
$$= 16x^2 - 50x + 39$$

$(2x-3)(2x-3)$
F O I L
- -

Q2. Solve the inequality $|2x - 5| > 1$ and give the answer in interval notation.

equality first $|2x-5| = 1$

$2x-5 = 1$ or $2x-5 = -1$

$2x = 6$ $\qquad 2x = 4$

key numbers $\longrightarrow$ $x = 3$ or $x = 2$

solutions are $(-\infty, 2) \cup (3, \infty)$.

Q3. Define the function $f(x) = \dfrac{2x+5}{x+3}$

(a) Compute $f(-4)$

(b) Use the three step precedure to find the inverse function $f^{-1}(x)$

(c) Use your formula from part (b) to compute $f^{-1}(3)$

$f(-4) = \dfrac{2(-4)+5}{(-4)+3} = \dfrac{-8+5}{-4+3} = \dfrac{-3}{-1} = \boxed{3}$

b) $y = \dfrac{2x+5}{x+3}$ so $(x+3)y = 2x+5$

$xy+3y = 2x+5$

$\underbrace{xy-2x}_{x(y-2)} = -3y+5$ ← divide both sides by $y-2$

and $x = \dfrac{-3y+5}{y-2}$

swap x, y

$y = \boxed{\dfrac{-3x+5}{x-2} = f^{-1}(x)}$

c) $f^{-1}(3) = \dfrac{-3(3)+5}{3-2}$

$= \dfrac{-9+5}{1} = \boxed{-4}$

as expected from part a).

Q4. Give the domains of these functions. Write the answers in interval notation.

(a) $f(x) = \dfrac{3}{x-4}$

(b) $g(x) = 2 + 3\sqrt{2x+6}$

x cannot be 4 in $f(x)$

so domain $= (-\infty, 4) \cup (4, \infty)$

b) need $2x+6 \geq 0$

so domain of $g(x)$ is $x \geq -3$

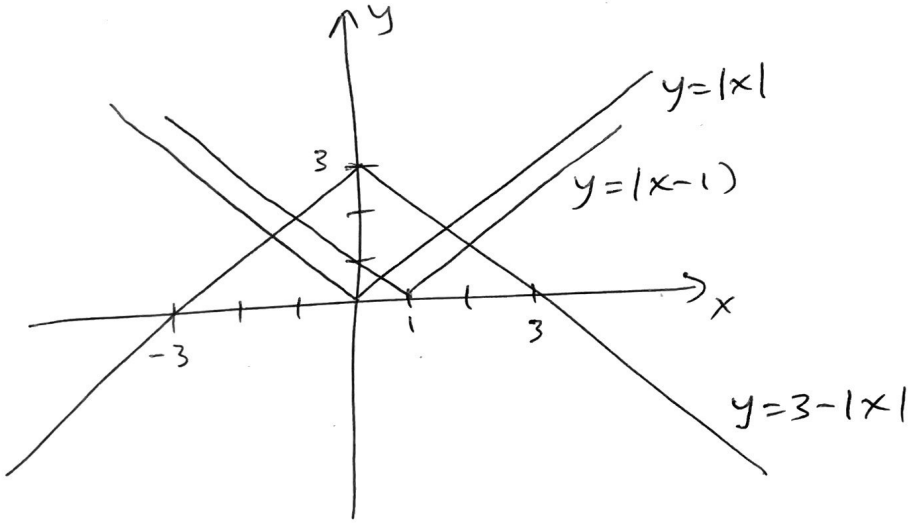
which is $[-3, \infty)$.

Q5.

(a) Carefully sketch the graph of the absolute value function $y = |x|$. Make sure to label and mark off numbers on the x and y axes.

(b) On the same axes draw the graph of $y = |x - 1|$ ← move right 1

(c) On the same axes also draw the graph of $y = 3 - |x|$ ← move up 3 and flip upside down.



Q6. Give the equation of the line that passes through the point $(3, 2)$ and is parallel to the line $4x + 2y = 19$.

find slope – put into slope-intercept form $y = mx + b$

$2y = -4x + 19$

$y = -2x + \frac{19}{2}$ so slope is -2

the line we want is parallel so must also have slope -2

$y = -2x + b$

use that $(3,2)$ is on line to find b

$2 = -2(3) + b$ $\quad b = 8$

$$\boxed{y = -2x + 8}$$

Q7. For the quadratic function $f(x) = -3x^2 - 6x + 5$ find the coordinates of its vertex.

formula for vertex (h,k) $\quad h = \frac{-b}{2a}$ $\quad k = f(h)$

$$h = \frac{-b}{2a} = \frac{-(-6)}{2(-3)} = \frac{6}{-6} = -1$$

$$k = f(-1) = -3(-1)^2 - 6(-1) + 5$$
$$= -3(1) - 6(-1) + 5$$
$$= -3 + 6 + 5 = 8$$

vertex at $(-1, 8)$

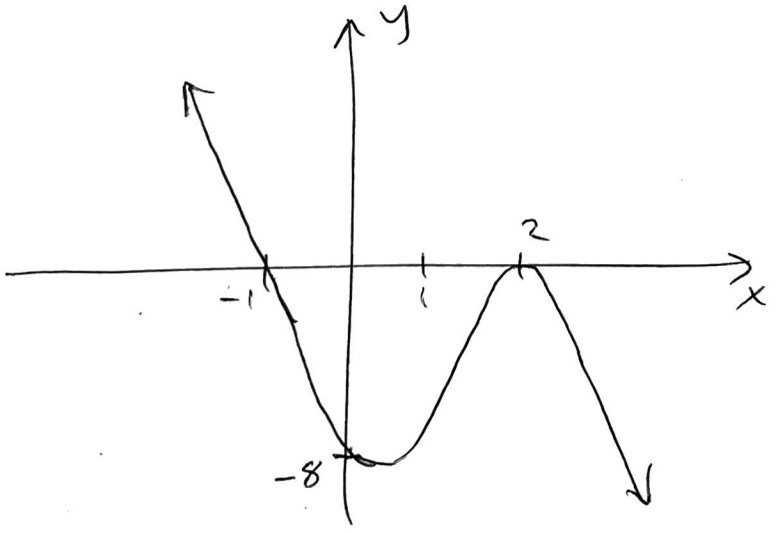
Q8. Let $g(x) = -2(x-2)^2(x+1)$.

(a) Find its end-behavior. Leading term is $-2x^3$ so ↑ ↘

(b) Find its x-intercepts. $x = 2$ with multiplicity 2 and $x = -1$

(c) Find its y-intercept. $g(0) = -8$

(d) Use parts (a), (b), (c) to sketch the graph of this polynomial function.



Q9. Let $h(x) = 2x^3 - x^2 - 5x - 2$.

(a) List all possible rational zeros of $h(x)$, according to the Rational Zeros Theorem.

(b) Find the three actual zeros.

rational zeros $= \pm \frac{\text{factors last}}{\text{factors first}} = \pm \frac{1,2}{1,2}$

$= \pm 1, \pm 2, \pm \frac{1}{2}$

test $k=1$ — not a zero

test $k=-1$

$-1 \mid 2 \quad -1 \quad -5 \quad -2$

$\qquad\quad -2 \quad 3 \quad 2$

$2 \quad -3 \quad -2 \quad \boxed{0}$ ✓

quotient

$h(x) = (x+1)(2x^2 - 3x - 2)$

ac method

$(2x+1)(x-2)$

actual zeros are

$\boxed{-1, 2, -\frac{1}{2}}$

Q10. A linear function has a graph that passes through the points $(3, -2)$ and $(-1, 4)$.

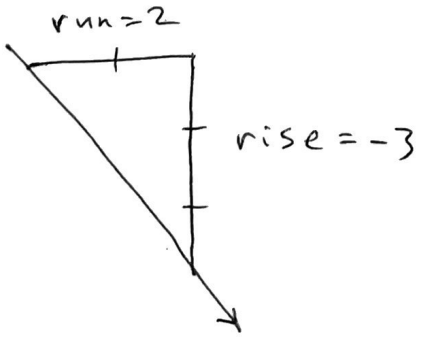
$x_1\ y_1 \quad x_2\ y_2$

(a) Find the slope of this line.

(b) Is this linear function increasing or decreasing?

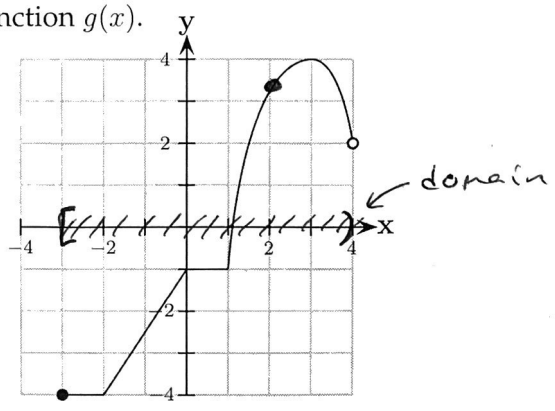
slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{(-1)-3} = \frac{4+2}{-1-3} = \frac{6}{-4}$

$= \boxed{-\frac{3}{2}}$



negative slope is always decreasing

Q11. This is the graph of the function $g(x)$.



Find the following:

(a) $g(2)$ = 3.3

(b) the domain of g $[-3, 4)$

(c) the range of g $[-4, 4]$

(d) the y-intercept -1

(e) any x-intercepts 1.2

Q12. For the graph of $g(x)$ in question 11, find

(a) all intervals where it is increasing $(-2, 0) \cup (1, 3)$

(b) all intervals where it is positive $(1.2, 4)$

(c) any x values where the function has a local minimum no local min

(d) any x values where the function has a local maximum 3

(e) Lastly, is g a one-to-one function? No

there are horizontal lines that hit the graph more than once, so it fails the horizontal line test.