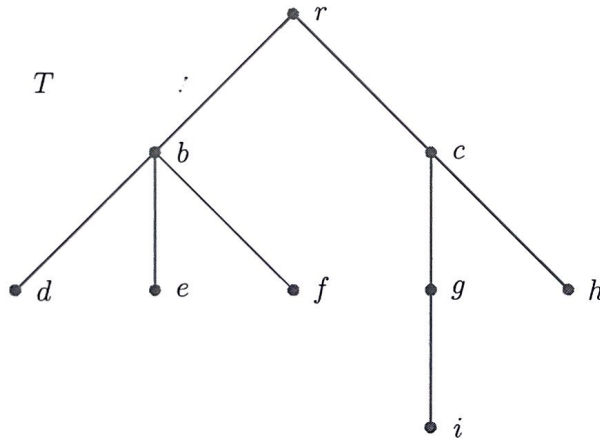


Put your phone away - it cannot be used during this exam. Do all eight questions in the space provided. They are worth 4 points each. Write your solutions clearly and neatly, showing any work you had to do to get the answer.

(1) List the vertices of this rooted tree T in

- (a) Inorder: d b e f r i g c h
 (b) Postorder: d e f b i g h c r



(2) For the same tree T above:

- (a) List the children of c . g h
 (b) List all the internal vertices. r b c g
 (c) Is this a binary tree? No
 (d) What is the height of this tree? 3
 (e) Is this tree balanced? Yes.

(3) Suppose a full 4-ary tree has 121 vertices.

(a) How many edges does it have? $121 - 1 = 120$ edges

(b) How many leaves does it have?

Show your work and explain your steps.

$$mi + 1 = 121 \quad m=4, \quad i = \text{number internal}$$

$$4i = 120$$

$$i = 30 \text{ internal}$$

$$\text{leaves} = 121 - 30 = 91$$

(4) The following expression involving single digit numbers is given in prefix notation:

$$* 2 - \uparrow 3 4 + 5 6$$

Calculate the number it is equal to. (Remember \uparrow means 'to the power of'.)

$$\begin{aligned} & * 2 - \underline{\uparrow 3 4} \underline{+ 5 6} \\ & = * 2 \underline{- 81 \quad 11} \\ & = * 2 \quad 70 \\ & = 140 \end{aligned}$$

$$3^4 = 81$$

$$5 + 6 = 11$$

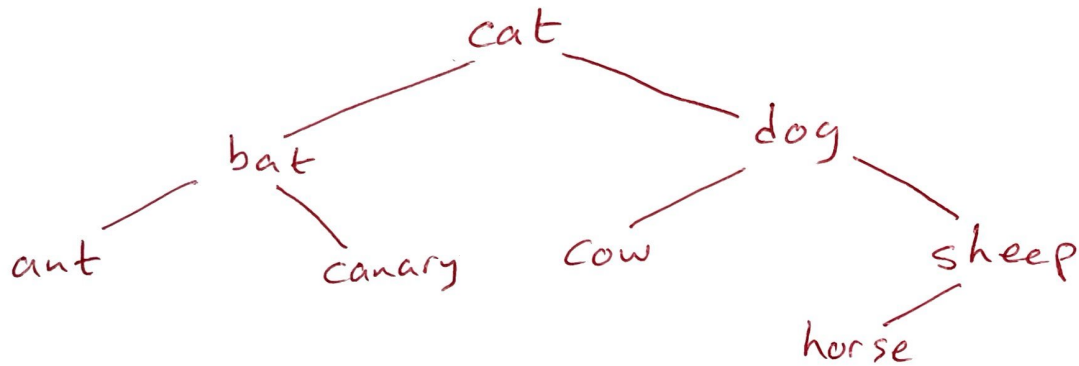
$$81 - 11 = 70$$

$$2 \cdot 70 = 140$$

(5) Construct a binary search tree for the words:

cat, dog, bat, cow, sheep, canary, horse, ant.

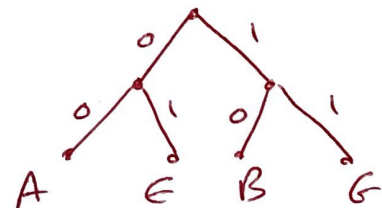
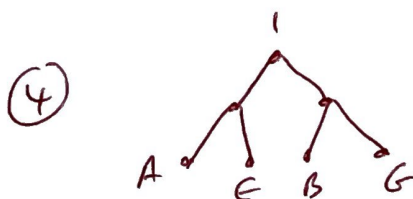
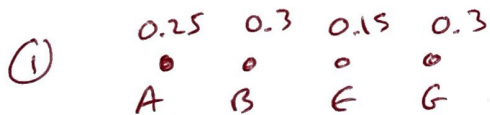
Use their alphabetical (dictionary) order.



(6) (a) Construct a Huffman code for the following letters with relative frequencies

A: 0.25, B: 0.3, E: 0.15, G: 0.3

(b) Use your code to encode the word AGE.



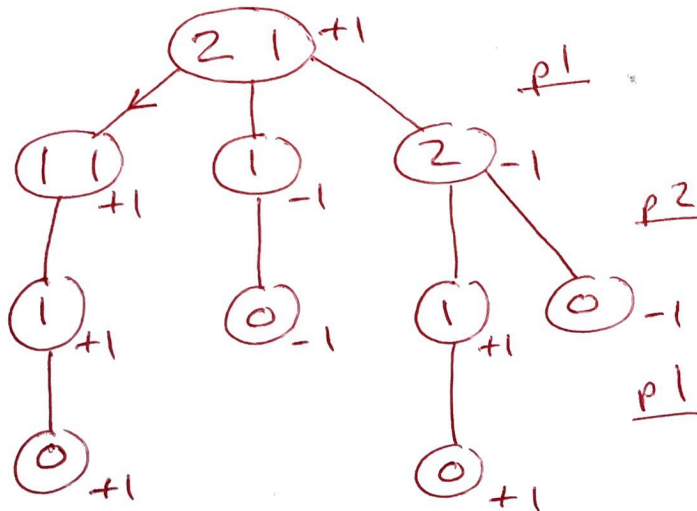
A	00
B	10
E	01
G	11

AGE: 00 11 01

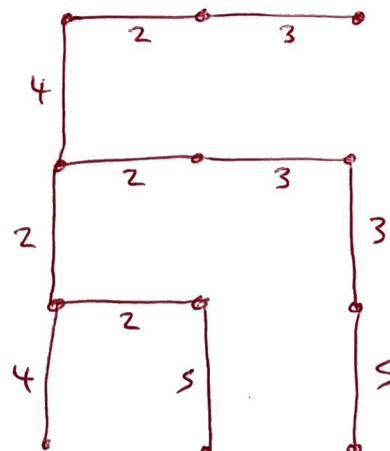
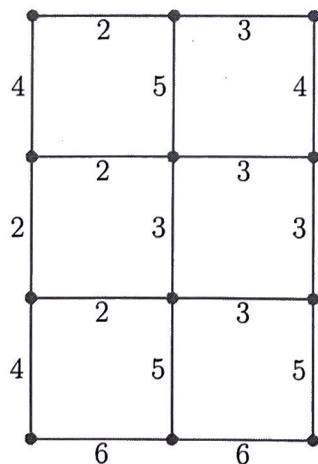
(7) In the game of Nim, a player may remove stones from one pile on their turn and the player taking the last stones wins.

(a) Draw the game tree for Nim played with 2 stones in the first pile and 1 stone in the second pile.

(b) What is a good first move for the first player? → Take one stone from pile of 2



(8) For the following weighted graph, find a minimum spanning tree and give the sum of the weights of the minimum spanning tree you find.



Kruskal

$$4(2) + 3(3) + 2(4) + 2(5) = 35 \quad \text{sum of weights}$$