CSI 35, Homework 6 on section 5.3, 5.4

Due by Wed, Mar 19.

Section 5.3 Recursion and structural induction

(1) Recall the recursive definition of the Fibonacci numbers from the notes. Use induction to prove that

$$f_1 + f_3 + \cdots + f_{2n-1} = f_{2n}$$

for all $n \geqslant 1$.

(2) Define the set T recursively with

Basis step: $(0, 0) \in T$

Recursive step: if $(a, b) \in T$ then $(a + 1, b - 2) \in T$ and $(a - 1, b + 1) \in T$.

(a) Give five different elements of T.

(b) Is $(0, -3)$ in T? Explain why or why not.

(3) Define the set U recursively with

Basis step: $(0, 0) \in U$

Recursive step: if $(a, b) \in U$ then $(a + 1, b - 1) \in U$ and $(a - 4, b + 4) \in U$.

Use structural induction to prove that if $(a, b) \in U$ then $a + b = 0$.

(4) Draw an example of a full binary tree with exactly 13 vertices.

(5) Our first example of a recursively defined function was $g(n)$ with

Basis step: $g(0) = 1$

Recursive step: $g(n + 1) = 2g(n) + n - 1$.

Use induction (regular induction or structural induction) to prove that

$$g(n) = 2^n - n \qquad \text{for} \qquad n \geqslant 0.$$

Section 5.4 Recursive Algorithms

(6) Trace through the recursive procedure *factorial*(n) from the notes when it is given $n = 5$ as input. Show every step the procedure uses to find $5!$. How many times does the procedure call itself?

(7) Remember that $x \bmod y$ means the remainder when you divide x by y.

(a) Why is $210 \bmod 5 = 0$?

(b) Use long division to find $397 \bmod 7$.

(8) Trace through the recursive procedure *gcd*(a, b) from the notes (the Euclidean algorithm) when it is given $a = 91$, $b = 119$ as input. Show every step and give the final answer.

(9) For any n can you work out what this recursive procedure outputs?

procedure fun(n : nonnegative integers)

if $n = 0$ **then return** 5

else return $3 \cdot$fun$(n - 1)$

(10) Trace through the recursive procedure *mergesort*(L) from the notes when

$$L = 6, 2, 4, 9, 1.$$

Show each step in detail and give the final output.

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes or section in the textbook.
- Ask me about it after class.
- Come to my office hours: Mon 2:00 - 3:00, Wed 2:00 - 3:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.